

Assignment 1, Econ270

Allan Timmermann

1 Stochastic Discount Factor in a simple Example Economy

You are in an economy with three states ($S = 3$) and three assets ($N = 3$). Consider the following Payoff matrix X :

$$D = \begin{bmatrix} 1 & 4 & 10 \\ 1 & 3 & 8 \\ 1 & 1 & 4 \end{bmatrix},$$

where the (s, n) 'th element of X gives the payoff in state s of the n th asset. The price vector associated with the three assets is given by $\mathbf{p} = (1 \ 3 \ 8)'$.

- (i) In this economy, what is \mathcal{X} ? Is the financial market complete?
- (ii) Does the price vector permit arbitrage trading strategies?
- (iii) Derive a formula for $x^*(s)$ and prove that $x^*(s)$ prices every x in \mathcal{X} .
- (iv) Show that in this economy x^* is not unique by constructing an example of another x^* , \tilde{x} , that gives the same prices for all payoffs x in \mathcal{X} as does x^* when used in the relation $p(x) = E[x^*x]$.

Traders in the economy want to price an option written on the second asset with an exercise price of 3. Hence the payoff on this option in state s is $x_{option} = \text{Max}[x_2(s) - 3, 0]$.

- (v) Does this option have the same expected value under the different x^* measures? What is the price of the option?

Now suppose that a fourth asset is added to the above list. This asset has a price of 1 and pays off $x_4 = (0, 0, 4)'$ in the three states.

- (vi) Derive $x^*(s)$ using the expanded set of assets.
- (vii) What is the price of the option in (v) under this expanded set of assets?
- (viii) Is the new measure, $x^*(s)$, in the expanded economy unique or can you still use the argument in (iv) to construct another \tilde{x} with the same pricing implications as x^* ?
- (ix) What is the Return space \mathcal{R}^* in the expanded economy?
- (x) What is the Excess return space \mathcal{R}^{e*} in the expanded economy?