

# Assignment 2, ECON 270

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## 1 Martingale Properties of Asset Prices

1. Consider a two-period  $(t, t + 1)$ , two-state economy satisfying the following pricing relationship between asset prices,  $P$ , consumption,  $C$ , and any dividends the assets may pay,  $D$ :

$$P_t = E_t[\beta \frac{U'(C_{t+1})}{U'(C_t)} (D_{t+1} + P_{t+1})],$$

where  $\beta = 1/1.05$ . Utility is given by

$$U(C) = C - 0.1C^2.$$

In equilibrium consumption equals dividends. At time  $t$ ,  $D_t = C_t = 1$  while at time  $t + 1$ , dividends can either grow to  $D_{t+1} = 4$  or to  $D_{t+1} = 2$ . Both states have the same probability, i.e.  $\pi(s_{t+1}) = 1/2$ .

(i) Consider a one-period discount bond with certain payoff  $B_{t+1} = 1$ . Work out the price of this bond,  $B_t$ , at time  $t$ .

(ii) Next, compute the risk-neutral probabilities,  $\pi^*(s_{t+1})$ ,  $s_{t+1} = 1, 2$ . How do these relate to the 'objective' probabilities,  $\pi(s_{t+1}) = 1/2$ ?

(iii) Consider the payoffs on a Call option written on the dividend stream at time  $t + 1$  with a strike price of 2, i.e. with payoff  $Call = Max(D_{t+1} - 2, 0)$ . Show how to price this call option using the risk-neutral probabilities. How does its price compare to its actuarial value (i.e. the simple expected payoff discounted at the risk-free rate)?

(iv) Show that the properly transformed Call option price,  $Call$ , follows a martingale in this economy.

## 2 Bond Prices

The Excel file `yielddata.xls` on the website for Econ270 contains monthly time-series of yields on US discount T-bonds from 1959-2000. You are asked to do the following:

(i) Characterize how the term-structure of interest rates has moved over time by comparing the 1, 6, 12, 24, 48 and 120 month yields. How often is the term-structure of interest rates downwardsloping?

(ii) Based on sample moments for the yield data, try to estimate the parameters of the simple Vasicek model. How well does the model fit the data?

(iii) Repeat part (ii) for the Cox-Ingersoll-Ross model. Does this model better capture features of the interest rate data? What are the obvious limitations of this model?

(iv) How would you test the Vasicek model more formally using term structure data?