

## Assignment 5, Econ270

1.

(i) Prove the put-call relation for American options:

$$G(S, t, T, X) \geq F(S, t, T, X) - S + XP(t, T),$$

where, as in the lectures,  $G(\cdot)$  is the value of an American put,  $F(\cdot)$  is the value of an American call,  $S$  is the stock price,  $X$  is the exercise price, and  $P(t, T)$  is the discount factor ( $P(t, T) \leq 1$ ). The proof should cover both the case with and without dividends paid on the underlying index.

(ii) Use a payoff table to prove the following relation: For  $X_2 > X_1$  (exercise prices),

$$f(\cdot, X_1) - f(\cdot, X_2) \leq P(t, T)(X_2 - X_1),$$

where  $f(\cdot)$  is the value of a European call option.

(iii) Prove that the value of a European put option is a convex function of its strike price.

2. Consider the following binomial model which gives the price of a stock at different points in time:

		121
	110	
100		100
	90.91	
		82.64
time 0	time 1	time 2

The return in the 'up' state ( $u$ ) is 10%, the return in the down state ( $d$ ) is -9.09%, and the risk-free interest rate is 4%. We are considering pricing American options that expire at time 2.

(i) Find the risk-neutral probability ( $p$ ) of an 'up' state. Briefly interpret this measure.

(ii) At each node find the price of a call option with an exercise price ( $K$ ) of 95. What is the fair value of the call option at time 0?

(iii) Derive the hedge ratio ( $h$ ) between the stock and the call option at the nodes at time 0 and time 1. Briefly explain why  $h$  increases when the stock price goes up.

(iv) Repeat (ii) and (iii) for a put option with an exercise price of 105.

(v) Establish a bound on the stock price such that if the price goes below it, then premature exercise of the put option considered in (iv) becomes optimal. Is this bound ever reached by the stock price in the binomial tree above?