Risk and Return Reaction of the Stock Market to News

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Abstract

This paper analyzes the reaction of stock returns to news about the state of the economy. We develop a general equilibrium asset pricing model where the investor learns about the growth rate of the economy through two sources of information, dividend realizations and regularly scheduled announcements about the state of the economy. We distinguish between dividend news and unexpected part of announcements and characterize the reaction of both level and conditional volatility of unexpected returns. The returns react quite differently to these two different sources of information whereas the reaction of conditional volatility is similar. Under certain assumptions, the reaction to dividend news is positive whereas the reaction to an external signal is negative and the conditional volatility reacts positively to both news. Our model is able to account for several empirical facts, such as time-varying expected returns and conditional volatility, time- and state-dependent reaction to news, the greater impact of news variables that are released earlier in a given month as well as the calm-before-the-storm effect.

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1 Introduction

There is no question that the reaction of stock prices to news is of central importance to financial decision making. Investors need to know how return dynamics are affected by additional information for portfolio allocation, risk management and pricing derivative securities among other important financial decisions. The reaction of stock prices to news might reveal important information on the link between fundamentals and prices for managers and policy makers as well as other financial market participants. Finally, the link between stock prices and news might shed light on one of the central concepts of modern finance, market efficiency.

The main problem in analyzing the reaction of stock prices to news is that it is relatively difficult to observe and distinguish the arrival of additional information. Furthermore, it is also relatively difficult to accurately measure the information content of an announcement. Scheduled news such as release of macroeconomic data provides a good starting point. First of all, the timing of these news is generally exogenously determined and known in advance by financial market participants. Secondly, it is relatively easy to quantify investors' expectations about scheduled announcements by employing either model- or survey-based measures. Hence, it is not surprising to find a large literature on the reaction of stock prices to macroeconomic announcements.

The literature mostly focuses on the reaction of an aggregate index rather than individual stock prices to macroeconomic news to establish the link between the stock market and fundamentals. In face of new information about the state of the economy, investors update their beliefs about fundamentals such as the growth rate of the economy and interest rates, which in turn affects the dynamics of stock returns. This fact that new information about macroeconomic variables affects not only the mean of returns on an aggregate index but also the conditional volatility is well documented in the finance literature.[1]

In contrast with the vast empirical evidence on the reaction of the stock market to news about macroeconomic variables, there still remain unanswered questions about the theoretical link between news about the state of the economy and the reaction of stock prices. A formal model is crucial not only for analyzing the theoretical link but also for constructing reasonable proxies for investors' expectations and uncertainty about an announcement. Instead of the current practice of using either ad hoc forecasting models or sur-

[1]A partial list of studies analyzing the reaction of stock prices to macroeconomic announcements includes McQueen and Roley (1993), Thorbecke (1997), Balduzzi, Elton, and Green (2001), Flannery and Protopapadakis (2002), Bomfim (2003), Guo (2004), Andersen, Bollerslev, Diebold, and Vega (2007), Bernanke and Kuttner (2005), Boyd, Hu, and Jagannathan (2005) and Gilbert (2009). Most of these studies focus on the reaction of an aggregate market index rather than individual stocks or portfolios with different characteristics with the exceptions of Thorbecke (1997), Guo (2004) and Bernanke and Kuttner (2005) who analyze the reaction to unanticipated changes in the target rate. Bernanke and Kuttner (2005) analyze the reaction of industry portfolios whereas Guo (2004) and Thorbecke (1997) analyze the reaction of portfolios formed on size.

veys, a formal model provides guidelines on how to construct such proxies for market expectations about announcements.

The contribution of this paper is to the theoretical literature on the link between news about the state of the economy and the reaction of the stock market.[2] In this paper, we analyze the reaction of stock returns to news about the state of the economy in a Lucas-type pure exchange economy with a representative agent (Lucas (1978)). Specifically, we first develop a general equilibrium asset pricing model where the investor learns about the growth rate of the economy through two sources of information, dividend realizations and regularly scheduled announcements about the state of the economy. In between announcement periods, the investor updates his beliefs about the current state of the economy through dividend realizations. On an announcement period, not only the growth rate of the economy can possibly change to a new level, but also the investor receives an external signal about the state of the economy in addition to the dividend realization. Differently from the previous literature, we distinguish between dividend news and the unexpected part of the external signal. In this framework, we then characterize the reaction of stock returns to news when there are only two possible states of the economy, low and high growth states. We first analyze the implications of our model for the level of returns and then we turn to the reaction of the conditional volatility.

The implications of our model for the reaction of stock returns to dividend news can be summarized as follows: The reaction of stock returns to dividends news depends on the magnitude of news variable itself unless the dividend growth process is equally volatile in both states. In other words, the reaction to dividend news is a nonlinear function of the news variable itself. Both the sign and magnitude of the reaction depend on unexpected dividend realizations. On announcement periods, the magnitude of the reaction to dividend news also depends on the unexpected part of the announcement whereas the sign is independent of the announcement. These implications of our model provide theoretical support for the empirical fact that the magnitude of the reaction to positive news is different than that to negative news of same importance.[3]

Under certain assumptions, the sign of the reaction to dividend news can be characterized unambiguously independent of investor's beliefs. Otherwise, the reaction can be positive or negative depending on dividend realization and investor's prior beliefs as well as the model parametrization. For example, if we assume that the investor is more risk averse than a log utility investor and that the dividend growth process is more volatile when the growth rate is low, then the reaction of stock returns is positive independent of

[2] Kim and Verrecchia (1991) develop a three-period partial equilibrium model to analyze the market reaction to anticipated announcements. They conclude that a price change reflects the change in investors' expectations due to the arrival of new information, whereas volume arises due to information asymmetries. Veronesi (1999) analyzes the reaction of the aggregate stock market to news about the growth rate of dividends and finds that stock prices overreact to bad news when the growth rate of dividends is high and underreact to good news when it is low. In a similar framework to ours, Veronesi (2000) analyzes the relation between stock returns and the quality of information and finds that higher quality information leads to an increase in the risk premium.

[3] Andersen, Bollerslev, Diebold, and Vega (2003)

investor's beliefs if the dividend realization is large enough. Although the sign of the reaction to dividend news can be characterized independent of investor's prior beliefs under these assumptions, the magnitude of the reaction always depends on investor's prior beliefs. These implications of our model provides theoretical support for the empirical fact that the reaction of stock returns to news depends on the state of the economy and investor's beliefs about it.[4]

There are two channels through which dividend news affects returns on the risky asset, its direct effect on investor's consumption and its indirect effect through investor's beliefs. The reaction of returns to dividend news through the first channel is always positive and equal to one. In other words, a one percent higher than expected dividend realization would result in a one percent increase in unexpected returns assuming that model parameters are such that the dividend realization does not affect investor's beliefs. The indirect effect of dividend news can be positive or negative depending on investor's coefficient of risk aversion and the dividend realization as well as other model parameters. For example, if the indirect effect of dividend news is positive, then the overall reaction to dividend news is also positive. However, if the indirect effect is negative, then which effect dominates depends on model parameters.

The reaction of returns to dividend news also depends on the number of periods till the next announcement. All else equal, a news variable released in an earlier period in between announcements will have a greater impact on investor's beliefs than a news variable of the same magnitude released in a later period. The implication for returns is that the reaction would be stronger to a dividend news released earlier, assuming that the reaction to this dividend news is positive. This result is due to the fact that the investor puts more weight on a news variable released earlier as he knows that the economy will be in the current state till the next announcement. In the extreme, any additional information about the state of the economy observed one period before the announcement would not be as useful since the investor knows that the economy might possibly switch to a new state in the following period on the announcement day. This implication of our model is inline with the empirical findings of Andersen, Bollerslev, Diebold, and Vega (2003) who argue that the explanatory power of macroeconomic variables released earlier in a given month is higher than that of variables released later.

On the other hand, the reaction to the external signal is relatively different than the reaction to dividend news as the only channel through which the external signal affects returns on the risky asset is through investor's beliefs. Hence, the sign of reaction to the external signal can be completely characterized without ambiguity. For example, if the investor is more risk averse than a log utility investor and the external signal is more volatile when the growth rate is low, then the reaction of stock returns to the unexpected part of the announcement is positive if and only if the external signal is large enough. Furthermore, if the precision of

[4] Boyd, Hu, and Jagannathan (2005) and Andersen, Bollerslev, Diebold, and Vega (2007)

the external signal is independent of the state of the economy, then the sign of the reaction to the external signal depends only on the investor's coefficient of risk aversion. If the investor is more risk averse than log utility, then the reaction is negative.

Whether it is dividend news or an external signal, any news about the state of the economy reveals information about two components of returns, expected future cash flows and the discount factor that investors use to discount these future cash flows. In a model with a representative investor with a power utility, a positive piece of information has two effects in equilibrium. First, the investor believes that the growth rate is higher than previously expected. Secondly, he believes that the discount factor is also higher than previously expected since the discount factor is a function of the investor's consumption which is equal to dividends in equilibrium. In this framework, which of these two effects dominates in equilibrium depends on whether the investor is more risk averse than a log utility investor. If the investor is more risk averse than a log utility investor, then the negative impact of a higher than expected discount rate on the price of the risky asset would dominate the positive impact of higher than expected future cash flows. For the external signal, the reaction would generally be negative to positive news if the investor is more risk averse than a log utility investor as the only effect of an external signal is on investor's beliefs. However, for dividend news, the reaction would generally depend on the model parametrization even if the investor is more risk averse than a log utility investor since the direct effect of dividend news on returns is positive and dominates the negative impact of the indirect effect under certain assumptions. These implications of our model provide theoretical support for recent empirical findings that the reaction of stock returns to a positive external signal is negative under certain states of the economy.[5]

The implications of our model for the reaction of conditional volatility can be summarized as follows: First of all, the conditional volatility can be expressed as an ARCH-type model under the assumptions of our model and it depends on the change in investor's beliefs and the associated uncertainty about the state of the economy. Similar to the reaction of level of returns to dividend news, the reaction of conditional volatility also depends of the magnitude of news variable itself unless the dividend growth process has the same volatility in both states. In other words, our model is capable of generating the asymmetric effect of news on conditional volatility. On announcement periods, the magnitude of reaction of conditional volatility to dividend news also depends on the unexpected part of the announcement whereas the sign of reaction is independent of the announcement.

Whether it is dividend news or external signal, there are two channels through which additional information affects the conditional volatility. The first channel is through the change in investor's beliefs. Any news variable that increases the probability assigned to the high growth state will result in an increase in condi-

[5] Boyd, Hu, and Jagannathan (2005)

tional volatility if and only if the dividend growth process is more volatile when the growth rate is high. The second channel is through investor's uncertainty about the state variable. Any additional information that increases investor's uncertainty will always result in an increase in conditional volatility. Which of these two effects dominates depends on investor's beliefs, news variable itself as well as the model parametrization. Under certain assumptions, the reaction of conditional volatility to dividend news or external signal can be completely characterized. For example, assuming that the dividend growth process (the external signal) is more volatile in the low growth state and that the investor assigns a higher probability to the high growth state after observing the dividend realization, the reaction of conditional volatility is positive if and only if the dividend realization (the external signal) is large enough.

The conditional volatility also changes deterministically with respect to the number of periods till the next announcement, all else equal. For the same set of investor's beliefs in two different periods, the conditional volatility is lower in the later period, i.e in the period closer to the next announcement. In other words, the unconditional level of volatility decreases as the announcement approaches. This is again due to the fact that the investor puts more weight on any information revealed in an earlier period in between announcements. In the extreme, the conditional volatility is higher on an announcement period than it is on the period just before the announcement. The sensitivity of conditional volatility to dividend news also decreases as the announcement gets closer. These implications suggest that our model can also account for the empirical fact that the conditional volatility of returns decreases significantly the day before an announcement, dubbed the "calm-before-the-storm" by Jones, Lamont, and Lumsdaine (1998).

The rest of the paper is organized as follows: Section 2 introduces the setup and assumptions of our model and presents analytical solutions for the return on the risky asset and its conditional volatility. Section 3.1 characterizes the reaction of level of returns to dividend news and external signal and discusses the empirical implications of our model. Section 3.2 characterizes the reaction of conditional volatility to these two news variable and discusses the empirical implications. Section 4 summarizes our findings and gives direction for future research.

2 The Model

In this section, we discuss the main assumptions as well as the information structure of the model. We consider a pure exchange economy (Lucas (1978)) in discrete time where a representative investor infers about the true state of the dividend growth rate through dividend realizations and external public signals that reveal additional information about the growth rate of dividends. The preferences of the representative investor in this economy are represented by a constant relative risk aversion utility over consumption,

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(C_t) & \text{if } \gamma = 1 \end{cases} \tag{1}$$

where C_t denotes investor's consumption in period t and γ is the coefficient of relative risk aversion. Investor's opportunity set consists of a risky asset whose supply is fixed and normalized to 1 and a riskless asset whose risk-free rate of return is r_t^f. Dividends of the risky asset at time t, D_t, grow according to the following process

$$\Delta d_t = \mu_{d,S_n} + \sigma_{d,S_n}\varepsilon_{d,t} \quad \text{for } T_{n-1} < t \leq T_n \tag{2}$$

where $d_t = \log(D_t)$ is the log-dividend at time t, Δ denotes the first difference operator (i.e. $\Delta d_t = d_t - d_{t-1}$) and $\varepsilon_{d,t}$ is an independently and identically distributed Gaussian random variable with zero mean and unit variance. S_n represents the true state of the dividend growth rate for the time period between T_{n-1} and T_n. We assume that the investor infers about the state of the dividend growth rate not only through dividend realizations but also through an imperfect external public signal, x_n, that is observed only on announcement periods,

$$x_n = \mu_{x,S_n} + \sigma_{x,S_n}\varepsilon_{x,T_n} \tag{3}$$

where ε_{x,T_n} is an independently and identically distributed Gaussian random variable with zero mean and unit variance and is independent of ε_{d,T_n}.

We assume that the state variable S_n can take on a finite number of values, i.e. $S_n \in \{1, 2, \ldots, N\}$. We further assume without loss of generality that $\mu_{d,1} > \mu_{d,2} > \ldots > \mu_{d,N}$ and $\mu_{x,1} > \mu_{x,2} > \ldots > \mu_{x,N}$. In the general setup of the model, we do not restrict the variance of the dividend growth rate, σ_{d,S_n}, and the variance of the external public signal, σ_{x,S_n} in different states. The state variable, S_n, follows a first-order N-state Markov chain where the transition probabilities are given by an $N \times N$ matrix $\mathbf{Q}$

$$\{\Pr(S_n = j | S_{n-1} = i)\} = \{q_{ij}\} = \mathbf{Q} \tag{4}$$

We assume that the state of the dividend growth process can possibly take on a new value according to the transition probability matrix only on announcement periods. In other words, S_n is realized on announcement period T_{n-1} and is the state of the dividend growth process until the next announcement period T_n when the investor observes the external signal x_n that reveals additional information about S_n. Hence, we use the index n to track the state variable and t to track the realizations of the dividend process. One should also note that T_n is not only the announcement period of the external public signal, x_n, but it is also the period where the dividend growth process possibly switches to a new state. For analytical tractability, we consider regularly scheduled announcements every T periods, i.e. $T_n - T_{n-1} = T$ for $n = 1, 2, \ldots$ and $T_0 = 0$.

In this setup, the investor never observes the true state of the dividend growth process. However, he infers about the state variable in between announcement periods by observing dividend realizations every period. On announcement periods, he not only receives an additional imperfect signal about the state variable but also knows that the state of the dividend growth process might change. The main advantage of our model is that not only is it analytically tractable and suitable for the question posed in this paper but it is also realistic. A special case our model where the representative investor observes weekly dividend realizations and quarterly public announcements would closely replicate the structure of information flow in financial markets where investors never observe the true growth rate of the economy in a given quarter. They infer about it through many signals such earnings and dividends realizations of individual companies during the quarter. At the end of the quarter (with a one month delay), investors receive additional signals such as the unemployment rate or the growth rate of GDP and update their beliefs.

Our model nests many other preceding models in the literature as special cases. The closest model to ours is that of Veronesi (2000) where he analyzes the effect of information quality on stock returns and equity premium. One can obtain the model of Veronesi (2000) as a special case of our model by assuming that the external signal is observed every period rather than on pre-specified announcement periods (i.e. by setting $T = 1$ and $\mu_{d,S_n} = \mu_{x,S_n}$ for all S_n in our model). This extension of the Veronesi's model is important as our model is better suited to analyze the reaction of stock returns to public announcements about the state of the economy and it replicates the structure of information flow in financial markets more closely. If we assume that there are only two possible states of the dividend growth process ($N = 2$), that the external signal is observed every period ($T = 1$) and that the dividend process and the external signal have the same mean for all states ($\mu_{d,S_n} = \mu_{x,S_n}$ for all S_n), then we obtain the model of Veronesi (1999) when the external signal does not reveal any information ($\sigma_{x,S_n} = \infty$ for $n = 1, 2$) and the model of Cecchetti, Lam, and Mark (1990) when the external signal reveals the true state ($\sigma_{x,S_n} = 0$ for $n = 1, 2$).

2.1 Investor's Belief

Our model is a general equilibrium asset pricing model with a representative investor learn about the dividend process. A model where investors update their beliefs in face of information is a natural choice for the question analyzed in this paper. Furthermore, asset pricing models with learning are known to generate dynamics such as time-varying volatility and expected returns due to changes in investor's beliefs. Hence, investor's beliefs play a central role in our model. In this section, we analyze how investor's beliefs evolve over time as new information about the state of the dividend growth process arrives.

Investor's beliefs about the current state of the dividend process depends on his current information set. As he receives additional information about the current state of the dividend process through dividend realizations and announcements, he updates his prior beliefs according to the Bayes' rule. Specifically, let π_{jt} denote the probability that he assigns to state j as being the value of the current state variable, S_n, given his information set $\mathcal{F}_t$ which includes past dividend realizations and announcements. Assuming that the investor has prior beliefs about the initial state of the dividend process at time 0 before observing any dividend realizations or announcements (π_{j0} for $j = 1, 2, \ldots, N$), then the following lemma characterizes investor's beliefs about the state variable:

Lemma 1. *The probability that the investor assigns to state j as being the value of the current state variable, S_n, at time t,* $\pi_{jt} = \Pr(S_n = j|\mathcal{F}_t) =$

$$\begin{cases} \sum_{i=1}^{N} \Pr(S_{n-1} = i|\mathcal{F}_{T_{n-1}}) q_{ij} & \text{if } t = T_{n-1} \\ \dfrac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}}) \pi_{j,t-1}}{\sum_{i=1}^{N} \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}}) \pi_{i,t-1}} & \text{if } T_{n-1} < t < T_n \qquad \text{for } n = 1, 2, \ldots \\ \dfrac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}}) \phi(\frac{x_n - \mu_{x,j}}{\sigma_{x,j}}) \pi_{j,t-1}}{\sum_{i=1}^{N} \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}}) \phi(\frac{x_n - \mu_{x,i}}{\sigma_{x,i}}) \pi_{i,t-1}} & \text{if } t = T_n \end{cases} \tag{5}$$

where $\phi(\cdot)$ is the standard normal density function and $\Pr(S_{n-1} = i|\mathcal{F}_{T_{n-1}})$ is similar to the third case and can be obtained by setting $t = T_{n-1}$ in the third case of Equation (5).

Proof. All proofs are in the appendix. □

Investor's beliefs need to be analyzed in three different cases. On the announcement period T_{n-1}, the investor updates his beliefs about the state of the dividend process since the last announcement period, $\Pr(S_{n-1} = i|\mathcal{F}_{T_{n-1}})$. Furthermore, the investor also knows that the dividend process might have switched to a new state according to the transition probability matrix $\mathbf{Q}$. Hence, he forms his beliefs about the current state of the dividend process, S_n, based on his beliefs about the past state variable S_{n-1} and the transition probability matrix. In between announcement periods T_{n-1} and T_n, the investor observes dividend realizations and updates his beliefs according to the Bayes' rule based on the law of motion of the dividend process. Finally, on the announcement period T_n, the investor receives an additional signal x_n about the current state and updates his beliefs according to the Bayes' rule using the information embedded in the dividend realization as well as the external signal. One should note that there are two probabilities associated with an announcement period, the first is the probability that the investor assigns to the state of the dividend growth process since the previous announcement period (the third case in Lemma 1) and the second is the probability that he assigns to the state of the dividend growth process until the next announcement period (the first case in Lemma 1).

The probability that the investor assigns to different states characterizes not only the investor's fluctuating expectations but also his uncertainty about the state of the dividend growth process. As we discuss in the next section, it is the investor's fluctuating expectations that generates dynamics in prices and returns that is not possible with standard models without learning.

2.2 Equilibrium Asset Prices and Returns

Equilibrium price process of the risky asset can be derived analytically from the investor's utility maximization problem. In this section, we derive closed-form expressions for the price of the risky asset on announcement and non-announcement periods. We then derive general expressions for unexpected log returns and its conditional volatility.

The investor chooses the fraction of wealth invested in the risky asset, α_t, and consumption, C_t, in order to solve the following maximization problem:

$$\max_{C_t,\alpha_t} E_t[\sum_{s=0}^{\infty} \beta^s U(C_{t+s})] \tag{6}$$

subject to the budget constraint:

$$W_{t+1} = \Big(W_t - C_t\Big)\Big(\alpha_t\Big(\frac{P_{t+1} + D_{t+1} - P_t}{P_t}\Big) + (1-\alpha_t)r^f_{t+1}\Big) \tag{7}$$

where W_t denotes investors' wealth at time t and P_t denotes the price of the risky asset. β is the investor's time impatience parameter and $E_t[\cdot]$ denotes expectation conditional on the available information at time t, $\mathcal{F}_t$. The Euler equation for the maximization problem is given by

$$P_t = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)}(P_{t+1} + D_{t+1})\right] \tag{8}$$

An equilibrium is defined by a vector process $(C_t, \alpha_t, P_t, r^f_t)$ such that the Euler equation in (8) holds and markets clear, i.e. $\alpha_t = 1$ and $C_t = D_t$.

In order to solve for the price of the risky asset, we first need to express the price-dividend ratio on announcement periods as a function of the true state variable. We then derive the closed-form expression for the price-dividend ratio of the risky asset as a function of the investor's beliefs about the state variable. Our solution approach is closest to those of Veronesi (2000) and Cecchetti, Lam, and Mark (1990). The following lemma characterizes the price-dividend ratio on announcement periods as a function of the true state variable assuming that the transversality condition for our model holds.[6]

Lemma 2. *Let λ_j denote the price-dividend ratio of the risky asset on an announcement period if the true state of the dividend process since the last announcement period is state j, then λ_j can be expressed as:*

$$\lambda_j = [(\mathbf{I} - \mathbf{HQ})^{-1}\mathbf{QG}]_j \geq 0 \quad \textit{for } j = 1, 2, \ldots, N$$

[6]The transversality condition for our model can be expressed as $\lim_{s\to\infty} E_t\left[\beta^s\left(\frac{D_{t+s}}{D_t}\right)^{-\gamma} P_{t+s}\right] = 0$. A necessary and sufficient condition for the transversality condition to hold is $\beta e^{a_i} < 1$ for $i = 1, 2, \ldots, N$ where a_i is defined in Lemma 2.

where the operator $[\cdot]_j$ refers to the j^{th} element of the vector. $\mathbf{I}$ *is the* $N \times N$ *identity matrix and* $\mathbf{Q}$ *is the transition probability matrix defined in Equation (4).* $\mathbf{G}$ *is a* $N \times 1$ *vector whose* i^{th} *element is* βe^{a_i} *and* $\mathbf{H}$ *is a* $N \times N$ *diagonal matrix whose* i^{th} *diagonal element is* $\sum_{k=1}^{N} \beta e^{a_k} q_{ik}$ *where* $a_i = (1-\gamma)\mu_{d,i} + (1-\gamma)^2 \sigma_{d,i}^2/2$.

Proof. All proofs are in the appendix. □

The lemma suggests the price-dividend ratio would take one of the N possible values if the investor observes the true state of the dividend process. A direct implication of the above lemma is that the price-dividend ratio on the current announcement period would take one of the N possible values if the signal is perfect and reveals the true state of the dividend process since the last announcement period. However, since we assume that the signal is imperfect and does not reveal the true state variable, the price-dividend ratio on any announcement period is a weighted average of the N possible values given by λ_j in Lemma 2 where the weights are investor's beliefs about the state variable. The following proposition characterizes the price of the risky asset on announcement and non-announcement periods as a function of investor's beliefs and the dividend realization.

Proposition 1. *The price of the risky asset at time t is given by:*

$$P_t = \sum_{j=1}^{N} k_{j\tau} \pi_{jt} D_t \qquad \text{for } T_{n-1} < t \leq T_n \text{ and } n = 1, 2, \ldots \tag{9}$$

where $\pi_{jt} = \Pr(S_n = j|\mathcal{F}_t)$ is the probability that the investor assigns to state j as being the value of the current state variable at time t and $\tau = T_n - t$ is the number of days till the next announcement and $k_{j\tau}$ is a time varying positive function and is given by the following equation if the state variable is j

$$k_{j\tau} = \frac{(\beta e^{a_j})^{\tau+1} - 1}{\beta e^{a_j} - 1} - 1 + (\beta e^{a_j})^{\tau} \lambda_j \qquad \text{for } \tau = 0, 1, 2, \ldots, T-1 \tag{10}$$

Proof. All proofs are in the appendix. □

The price-dividend ratio of the risky asset in between announcements fluctuates with respect to changes in investor's beliefs about the current state of the dividend growth process as he receives additional information through dividend realizations. The price-dividend ratio changes also deterministically as a function of time until the next announcement. In an extreme case where the investor knows the true state until the next announcement, the price-dividend ratio monotonically approaches to one of the N values described in Lemma 2. The following proposition characterizes the unexpected log return on the risky asset and its conditional volatility.

Proposition 2. *For t such that $T_{n-1} < t \leq T_n$ and $n = 1, 2, \ldots$, let r_t denote the log return on the risky asset at time t (i.e. $r_t = \log(\frac{P_t + D_t}{P_{t-1}})$), then the unexpected log return on the risky asset at time t, r_t^* , is given by:*

$$r_t^* = r_t - E_{t-1}[r_t] = \frac{1}{1+\overline{\lambda}} \sum_{j=1}^{N} k_{j,\tau}(\pi_{jt} - \pi_{j,t-1}) + \Delta d_t - \sum_{j=1}^{N} \mu_{d,j}\pi_{j,t-1} \tag{11}$$

where $\overline{\lambda}$ is the long-term average price-dividend ratio and is given by $\overline{\lambda} = E[P_t/D_t] = (1/T)\sum_{j=1}^{N}\Omega_j \sum_{\tau=0}^{T-1} k_{j\tau}$ and $[\Omega_1, \Omega_2, \ldots, \Omega_N]$ is the stationary distribution vector of the transition probability matrix **Q**.

The conditional volatility of unexpected log return at time $t+1$ given information set at time t is given by

$$\begin{aligned} var_t(r_{t+1}^*) &= \sum_{j=1}^{N} \left(\left(\frac{k_{j,\tau-1}}{1+\overline{\lambda}} + \mu_{d,j} \right)^2 + \sigma_{d,j}^2 \right) \pi_{j,t} \\ &\quad - \left(\sum_{j=1}^{N} \left(\frac{k_{j,\tau-1}}{1+\overline{\lambda}} + \mu_{d,j} \right) \pi_{j,t} \right)^2 \end{aligned} \tag{12}$$

where $\tau = T_n - t$ and $k_{j,-1} = k_{j,T-1}$.

Proof. All proofs are in the appendix. □

The unexpected returns on the risky asset depend on the current dividend realization, investor's current beliefs and prior beliefs about the state variable. Hence, unexpected returns will not only react to dividend realizations but also to changes in investor's beliefs about the state variable. On the other hand, the conditional volatility is a nonlinear function of investor's current beliefs after having observed the current dividend realization.

To gain further intuition about the unexpected return and its conditional volatility, the following corollary characterizes the law of motions when there are only two possible states of the dividend growth process.

Corollary 1. *Assume that there are only two possible states of the dividend growth process, i.e. $N = 2$, then the unexpected return on the risky asset at time t such that $T_{n-1} < t \leq T_n$ and $n = 1, 2, \ldots$ is given by:*

$$r_t^* = \frac{1}{1+\overline{\lambda}}(k_{1,\tau} - k_{2,\tau})(\pi_{1,t} - \pi_{1,t-1}) + \Delta d_t - (\mu_{d,1}\pi_{1,t-1} + \mu_{d,2}(1 - \pi_{1,t-1})) \tag{13}$$

The conditional volatility of unexpected returns on the risky asset at time t is given by:

$$\begin{aligned} var_t(r_{t+1}^*) &= \sigma_{d,2}^2 \left(1 - \left(\frac{m_{1,\tau-1} - m_{2,\tau-1}}{m_{1,\tau} - m_{2,\tau}} \right)^2 \right) + \left(\frac{m_{1,\tau-1} - m_{2,\tau-1}}{m_{1,\tau} - m_{2,\tau}} \right)^2 var_{t-1}(r_t^*) \\ &\quad + (\sigma_{d,1}^2 - \sigma_{d,2}^2) \left(\pi_{1,t} - \left(\frac{m_{1,\tau-1} - m_{2,\tau-1}}{m_{1,\tau} - m_{2,\tau}} \right)^2 \pi_{1,t-1} \right) \\ &\quad + (m_{1,\tau-1} - m_{2,\tau-1})^2 (\pi_{1,t}(1 - \pi_{1,t}) - \pi_{1,t-1}(1 - \pi_{1,t-1})) \end{aligned} \tag{14}$$

where $m_{j,\tau} = \mu_{d,j} + k_{j,\tau}/(1+\overline{\lambda})$ *for* $\tau = -1, 0, 1, \ldots, T-1$ *and* $\tau = T_n - t$.

Unexpected returns on the risky asset reacts to two factors. The first is the dividend news defined as the difference between the realized dividend growth rate, Δd_t, and the expected dividend growth rate, $\mu_{d,1}\pi_{1,t-1} + \mu_{d,2}(1-\pi_{1,t-1})$. The second is the change in the investor's beliefs due to additional information received at time t. The second factor depends on the dividend news on non-announcement days and on both the dividend news and the unexpected part of the announcement on announcement days. As we discuss in the following section, the distinction between news observed from dividend realizations and those observed from announcements is important. News observed from dividend realizations affects the returns through two channels, its direct effect through investor's consumption and its indirect effect through investor's beliefs, whereas the effect of the announcement on returns is only through its effect on investor's beliefs.

Corollary 1 shows that our model is able to generate volatility clustering in returns. The conditional volatility of returns follows an ARCH-type process where the current conditional volatility of returns depends on the conditional volatility in the previous period. The corollary also shows that the conditional volatility depends on the changes in investor's beliefs and the associated uncertainty. The third term in Equation (14) is a function of the change in investor's beliefs due to additional information and the fourth term is the change in investor's uncertainty about the state variable. Hence, the conditional volatility reacts to changes in investor's beliefs as well as to changes in his uncertainty. These results are similar to the ones discussed in Veronesi (1999) and Veronesi (2000).

3 The Reaction of Stock Returns to News

In this section, we analyze the reaction of unexpected stock returns and its conditional volatility to news about the dividend growth rate under the assumptions of our model. We analytically derive the implications of the model where there are only two possible states of the dividend growth process.

We first distinguish between news from two different sources of information, dividend realizations and announcements. Dividend realizations affect not only investor's beliefs about the state variable but also his consumption whereas announcements only affect his beliefs. Hence, it is important to make this distinction as the reaction of stock returns to these news variable would be different. We denote dividend news by $u_{d,t}$ defined as the unexpected dividend realization at time t:

$$u_{d,t} = \Delta d_t - \overline{\mu}_{d,t-1} \tag{15}$$

where $\overline{\mu}_{d,t-1} = \sum_{j=1}^{N} \mu_{d,j}\pi_{j,t-1}$ is the expected growth rate of dividends based on investor's belief at time $t-1$. Similarly, news observed from the n^{th} announcement released at time T_n is defined as the unexpected part of the announcement:

$$u_{x,T_n} = x_n - \overline{\mu}_{x,T_n-1} \tag{16}$$

where $\overline{\mu}_{x,T_n-1} = \sum_{j=1}^{N} \mu_{x,j}\pi_{j,T_n-1}$ is the expected part of the announcement based on investor's beliefs at time $T_n - 1$. We should note that, by definition, news due to announcements (u_{x,T_n}) is observed only on announcement days every T periods whereas news due to dividend realizations ($u_{d,t}$) is observed every period. Hence, the stock price reacts to dividend realizations on non-announcement days and reacts to both dividend realizations and announcements on announcement days.

3.1 The Reaction of Level of Stock Returns to News

We first analyze the reaction of level of stock returns to news. The following proposition characterizes the reaction of stock returns to dividend news and announcements when there are only two possible states of the dividend process.

Proposition 3. *Assume that there are only two possible states of the dividend process, i.e.* $N = 2$, *then the sensitivity of unexpected returns to dividend news on non-announcement days is given by:*

$$\partial r_t^*/\partial u_{d,t} = 1 + f_1(u_{d,t}, \pi_{1,t-1})\left(\frac{k_{2,\tau} - k_{1,\tau}}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2\sigma_{d,2}^2}\right)\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}_{d,t-1}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right) \tag{17}$$

for $T_{n-1} < t < T_n$ *and* $n = 1, 2, \ldots$ *and the sensitivity of unexpected returns to dividend news on announcement days is given by:*

$$\partial r_{T_n}^*/\partial u_{d,T_n} = 1 + f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\left(\frac{\lambda_2 - \lambda_1}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{\sigma_{d,1}^2\sigma_{d,2}^2}\right)\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}_{d,T_n-1}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2}\right) \tag{18}$$

$n = 1, 2, \ldots$

The sensitivity of the unexpected returns to the unexpected part of the announcement is given by:

$$\partial r_{T_n}^*/\partial u_{x,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\left(\frac{\lambda_2 - \lambda_1}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{x,2}^2 - \sigma_{x,1}^2}{\sigma_{x,1}^2\sigma_{x,2}^2}\right)\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\overline{\sigma}_{x,T_n-1}^2}{\sigma_{x,2}^2 - \sigma_{x,1}^2}\right) \tag{19}$$

for $n = 1, 2, \ldots$ *and* $\overline{\sigma}_{d,t-1}^2 = \sigma_{d,1}^2\pi_{1,t-1} + \sigma_{d,2}^2(1-\pi_{1,t-1})$ *and* $\overline{\sigma}_{x,T_n-1}^2 = \sigma_{x,1}^2\pi_{1,T_n-1} + \sigma_{x,2}^2(1-\pi_{1,T_n-1})$. *The functions* $f_1 : \mathbb{R} \times [0,1] \to [0, 0.25]$ *and* $f_2 : \mathbb{R}^2 \times [0,1] \to [0, 0.25]$ *are real-valued positive functions bounded above by* 0.25 *and are defined in the appendix.*

Proof. All proofs are in the appendix. □

Proposition 3 shows that our model is able to generate time- and state-dependent reaction to dividend news as well as to unexpected part of announcements. In other words, our model is able to explain the state-dependent reaction of the aggregate stock market index to news variables such as employment numbers observed in the data. On non-announcement days, the sign and magnitude of the reaction to dividend news depend on the number of periods till the next announcement, the magnitude of dividend news itself and investor's beliefs prior to observing the current dividend realization. On announcement days, the sign and magnitude of the reaction to dividend news also depend on the unexpected part of the announcement. On the other hand, dividend news affects only the magnitude of the reaction to the announcement but not the sign which depends on investor's prior beliefs about the state variable as well as the unexpected part of the announcement. Proposition 3 also shows that the sign of reaction to dividend news or to announcements depends on the news variable itself. In other words, our model is able to explain the empirical observation that the reaction of stock returns to positive news is different than to negative news of similar magnitude. Under certain parameter restrictions and assumptions, we can characterize the reaction of unexpected returns to news variables independent of investor's beliefs and the number of periods till the next announcement. The following proposition characterizes the sign of the reaction of unexpected returns on the risky asset to dividend news.

Proposition 4. *(a) Assume that the representative agent is more risk averse than a log-utility investor, i.e.* $\gamma > 1$*, and the dividend growth process is more volatile in the low growth state, i.e.* $\sigma_{d,2} > \sigma_{d,1}$*, or equivalently assume that the representative agent is less risk averse than a log-utility investor, i.e.* $\gamma < 1$*, and the dividend growth process is more volatile in the high growth state, i.e.* $\sigma_{d,2} < \sigma_{d,1}$*; then the reaction of unexpected returns to dividend news is positive independent of the investor's beliefs if the dividend realization is large enough, i.e.* $\Delta d_t > \Delta d^*$*, where* $\Delta d^* = (\sigma_{d,2}^2 \mu_{d,1} - \sigma_{d,1}^2 \mu_{d,2})/(\sigma_{d,2}^2 - \sigma_{d,1}^2)$*. Otherwise, the sign of the reaction can be positive or negative depending on the investor's beliefs and dividend realizations.*

(b) Assume that the representative agent is less risk averse than a log-utility investor, i.e. $\gamma < 1$*, and the dividend growth process is more volatile in the low growth state, i.e.* $\sigma_{d,2} > \sigma_{d,1}$*, or equivalently assume that the representative agent is more risk averse than a log-utility investor, i.e.* $\gamma > 1$*, and the dividend growth process is more volatile in the high growth state, i.e.* $\sigma_{d,2} < \sigma_{d,1}$*; then the reaction of unexpected returns to dividend news is positive independent of the investor's beliefs if the dividend realization is small enough, i.e.* $\Delta d_t < \Delta d^*$*. Otherwise, the sign of the reaction can be positive or negative depending on the investor's beliefs and dividend realizations.*

(c) Assume that the representative agent is more risk averse than a log-utility investor, i.e. $\gamma > 1$*, the dividend growth process has the same volatility in both states, i.e.* $\sigma_{d,2} = \sigma_{d,1}$*, then the reaction of unexpected returns to dividend news is positive independent of the investor's beliefs and dividend realizations if* $(k_{1,T_n-t} - k_{2,T_n-t})/(1+\overline{\lambda}) > -4\sigma_{d,1}^2/(\mu_{d,1} - \mu_{d,2})$*. Otherwise, the sign of the reaction can be positive or*

negative depending on the investor's beliefs and dividend realizations.

(d) Assume that the representative agent is less risk averse than a log-utility investor, i.e. $\gamma < 1$*, and the dividend growth process has the same volatility in both states, i.e.* $\sigma_{d,2} = \sigma_{d,1}$*, then the reaction of unexpected returns to dividend news is positive independent of the investor's beliefs and dividend realizations.*

(e) Assume that the representative agent is a log-utility investor, i.e. $\gamma = 1$*, then the reaction of unexpected returns to dividend news is always positive and a one percent higher than expected dividend realization will result in a one percent increase in unexpected returns independent of investor's prior beliefs.*

Proof. All proofs are in the appendix. □

There are two channels through which dividend news affects the unexpected returns on the risky asset. The first channel is its direct effect on investor's consumption through the dividend realization. The effect of dividend news through the first channel is always equal to one, the first term in Equation (17) and (18). The second channel is its indirect effect through investor's beliefs, the second term in Equation (17) and (18). The indirect effect of dividend news can be positive or negative depending on investor's coefficient of risk aversion and the current dividend realization as well as other model parameters. If the indirect effect of dividend news is positive, then the reaction of unexpected returns would be positive. However, if the indirect effect of dividend news is negative, then the sign of the overall reaction depends on the relation between the direct and indirect effects of dividend news on unexpected returns. More specifically, if the magnitude of the indirect effect in absolute value is less than one, then the overall reaction to dividend news would be positive. Otherwise, the reaction to dividend news would be negative. Proposition 4 characterizes the regions of the dividend growth process for which the reaction of unexpected returns to dividend news is unambiguous. In other regions of the dividend growth process, the reaction can be positive or negative depending on investor's beliefs, dividend realization and model parametrization. Another interpretation of Proposition 4 is with respect to the dividends news rather than the dividend realization. For example, in case (a) of Proposition 4, the reaction of unexpected returns to dividend news would be unambiguously positive if the observed dividend news is large enough which is of course not independent of the investor's beliefs. Hence, Proposition 4 shows that the sign of the reaction depends on the news variable itself. Although the sign of the reaction to dividend news depends on several factors, the reaction is positive for a wide range of parameter choices.

On the other hand, the unexpected part of the announcement affects unexpected returns only through its effect on investor's beliefs. Hence, the sign of the reaction can be completely characterized as a function of the announcement independent of investor's beliefs prior to observing the announcement. The following proposition characterizes the sign of the reaction of unexpected returns on the risky asset to the unexpected part of the announcement.

Proposition 5. *(a) Assume that the representative agent is more risk averse than a log-utility investor, i.e.* $\gamma > 1$*, and the external signal has a higher volatility in the low growth state of the dividend process, i.e.* $\sigma_{x,2} > \sigma_{x,1}$*, or equivalently assume that the representative agent is less risk averse than a log-utility investor, i.e.* $\gamma < 1$*, and the external signal has a higher volatility in the high growth state of the dividend process, i.e.* $\sigma_{x,2} < \sigma_{x,1}$*; then the reaction of unexpected returns to the unexpected part of the announcement is positive independent of the investor's beliefs if the external signal is large enough, i.e.* $x_n > x^*$ *where* $x^* = (\sigma^2_{x,2}\mu_{x,1} - \sigma^2_{x,1}\mu_{x,2})/(\sigma^2_{x,2} - \sigma^2_{x,1})$*. Otherwise, the sign of the reaction is negative independent of the investor's beliefs.*

(b) Assume that the representative agent is more risk averse than a log-utility investor, i.e. $\gamma > 1$*, and the external signal has the same volatility in both states, i.e.* $\sigma_{x,2} = \sigma_{x,1}$*, then the reaction of unexpected returns to the unexpected part of the announcement is negative independent of the investor's beliefs and the announcement.*

(c) Assume that the representative agent is less risk averse than a log-utility investor, i.e. $\gamma < 1$*, and the external signal has the same volatility in both states, i.e.* $\sigma_{x,2} = \sigma_{x,1}$*, then the reaction of unexpected returns to the unexpected part of the announcement is positive independent of the investor's beliefs and the announcement.*

(d) Assume that the representative agent is a log-utility investor, i.e. $\gamma = 1$*, then unexpected returns do not react to the announcement independent of investor's prior beliefs.*

The effect of the announcement on unexpected returns is similar to the indirect effect of dividend news. Hence, the reaction to unexpected part of the announcement can be positive or negative depending on investor's coefficient of risk aversion and the announcement as well as other model parameters. Differently from dividend news, the sign of the reaction to announcement can be completely determined without any ambiguity as the only channel through which the external signal affects the unexpected returns is through investor's beliefs. Proposition 5 characterizes the values of the announcement for which the reaction to the unexpected part of the announcement is positive. There is also another interpretation of Proposition 5 similar to the one discussed above for Proposition 4 which suggests that the sign of the reaction to the announcement can be positive or negative depending on the unexpected part of the announcement.

Whether it is dividend news or an external signal, any news about the state of the dividend process reveals information about two components of returns, expected future dividends and the discount factor that the investor uses to discount future dividends. In a model with a representative investor with a power utility, a positive piece of information about the state of the dividend growth process has two effects in equilibrium. First, the investor believes that the growth rate of dividends is higher than previously expected. Secondly, he believes that the discount factor is also higher than previously expected since the discount factor is a function of the investor's consumption which is equal to dividends in equilibrium. In this framework, which of these two effects dominates in equilibrium depends on whether the investor is more or less risk averse than a log-utility investor. If the investor is more risk averse than a log utility investor, then the negative impact of a higher than expected discount rate on the price of the risky asset would dominate the positive impact of higher than expected future dividends, hence the negative reaction to positive news.

To gain further intuition into the negative reaction of stock returns to positive news, one can use a two-period model with a power utility. An unanticipated higher growth rate has two effects in equilibrium. The first effect is the income effect. An unanticipated good news about the growth rate results in a higher endowment in the second period. Investors are willing to pay more for the risky asset which is a claim on the second period consumption since the payoff is higher than previously expected. Hence, the income effect increases the current equilibrium price of the risky asset. The second effect is the substitution effect. Investors are willing to consume more in the current period due to a higher than expected consumption in the second period. In a power utility framework, a higher endowment in the second period increases the stochastic discount factor. Therefore, investors are discounting future payoffs at a higher rate. Hence, the substitution effect decreases the current equilibrium price of the risky asset. Which effect dominates in equilibrium depends on investors' risk aversion parameter, γ. If investors are more risk averse than a log-utility investor, i.e. $\gamma > 1$, the substitution effect dominates the income effect and the equilibrium asset price decreases. The opposite holds when $\gamma < 1$. If investors have a log utility (i.e. $\gamma = 1$), income and substitution effects cancel out, hence any news that just affects investor's beliefs does not have any effect on unexpected returns.

Although the sign of the reaction to dividend news can be characterized under few assumptions, the same is not true for the magnitude which depends on many factors including investor's coefficient of risk aversion, the number of periods till the next announcement and investor's prior beliefs as well as all other model parameters. The magnitude of the reaction can be characterized as functions of these factors under several assumptions. However, we focus here only on one of these factors, the number of periods till the next announcement. It is a special feature of our model and has not been previously analyzed in the literature. The following proposition characterizes the magnitude of the reaction of unexpected returns to dividend news as a function of time till the next announcement.

Proposition 6. *(a) Assume that the investor is not a log-utility investor, i.e.* $\gamma \neq 1$ *and that the reaction of unexpected returns to dividend news in periods* t_1 *and* t_2 *is positive (negative), then the magnitude of the reaction in absolute value in period* t_1 *is greater (smaller) than that of the reaction in period* t_2 *if and only if* $t_2 > t_1$*, all else equal.*

(b) Assume that the investor is a log-utility investor, i.e. $\gamma = 1$*, then the reaction of unexpected returns to dividend news does not depend on the number of periods till the next announcement.*

Proof. All proofs are in the appendix. □

Proposition 6 shows that the magnitude of the reaction depends on whether a news variable is observed earlier in between announcement days or later. Assuming that the model parameters are such that the reaction to a dividend news is positive, then the reaction would be stronger the earlier it is observed all else equal. This effect is a direct consequence of the effect of dividend news on investor's beliefs. Independent of the sign of the overall reaction, a news variable released earlier in between announcement days will have a greater impact on unexpected returns through its indirect effect on investor's beliefs than a news variable of the same magnitude released later given that the investor's prior beliefs are the same. The investor puts more weight on a news variable released earlier when updating his beliefs as he knows that the dividend process will be in the current state till the next announcement. Hence, any additional information about the state variable would be more important if it is revealed earlier in between announcement days. In the extreme, dividend news observed one period before the announcement would not be as useful as the investor knows that the dividend process might change to a new state the following period on the announcement day. Proposition 6 provides theoretical support for empirical findings of Andersen, Bollerslev, Diebold, and Vega (2003) who argue that the explanatory power of macroeconomic variables released earlier in a given month is higher than that of variables released later.

3.2 The Reaction of Conditional Volatility of Stock Returns to News

The reaction of conditional volatility of stock returns to news can be analyzed in a similar fashion. The following proposition characterizes the reaction of conditional volatility of stock returns to dividend news and announcements when there are only two possible states of the dividend process.

Proposition 7. *Assume that there are only two possible states of the dividend process, i.e.* $N = 2$*, then the sensitivity of conditional volatility of unexpected returns to dividend news on non-announcement days is* *given by:*

$$\partial var_t(r^*_{t+1})/\partial u_{d,t} = f_1(u_{d,t}, \pi_{1,t-1})\Big((\sigma^2_{d,1} - \sigma^2_{d,2}) + (m_{1,\tau-1} - m_{2,\tau-1})^2(1-2\pi_{1,t})\Big) \left(\frac{\sigma^2_{d,2} - \sigma^2_{d,1}}{\sigma^2_{d,1}\sigma^2_{d,2}}\right)\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2} - \sigma^2_{d,1}}\right) \tag{20}$$

for $T_{n-1} < t < T_n$ *and* $n = 1, 2, \ldots$ *and the sensitivity of unexpected returns to dividend news on announcement days is given by:*

$$\partial var_{T_n}(r^*_{T_n+1})/\partial u_{d,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\Big((\sigma^2_{d,1} - \sigma^2_{d,2}) + (m_{1,T-1} - m_{2,T-1})^2(1-2\pi_{1,T_n})\Big) \left(\frac{\sigma^2_{d,2} - \sigma^2_{d,1}}{\sigma^2_{d,1}\sigma^2_{d,2}}\right)\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}^2_{d,T_n-1}}{\sigma^2_{d,2} - \sigma^2_{d,1}}\right) \tag{21}$$

$n = 1, 2, \ldots$

The sensitivity of the unexpected returns to the unexpected part of the announcement is given by:

$$\partial var_{T_n}(r^*_{T_n+1})/\partial u_{x,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\Big((\sigma^2_{d,1} - \sigma^2_{d,2}) + (m_{1,T-1} - m_{2,T-1})^2(1-2\pi_{1,T_n})\Big) \left(\frac{\sigma^2_{x,2} - \sigma^2_{x,1}}{\sigma^2_{x,1}\sigma^2_{x,2}}\right)\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\overline{\sigma}^2_{x,T_n-1}}{\sigma^2_{x,2} - \sigma^2_{x,1}}\right) \tag{22}$$

for $n = 1, 2, \ldots$ *and* $\overline{\sigma}^2_{d,t-1} = \sigma^2_{d,1}\pi_{1,t-1} + \sigma^2_{d,2}(1-\pi_{1,t-1})$ *and* $\overline{\sigma}^2_{x,T_n-1} = \sigma^2_{x,1}\pi_{1,T_n-1} + \sigma^2_{x,2}(1-\pi_{1,T_n-1})$. *The functions* $f_1 : \mathbb{R} \times [0,1] \to [0, 0.25]$ *and* $f_2 : \mathbb{R}^2 \times [0,1] \to [0, 0.25]$ *are real-valued positive functions bounded above by* 0.25 *and are defined in the appendix.*

Proof. All proofs are in the appendix. □

Proposition 7 shows that our model is able to generate time- and state-dependent reaction of conditional volatility to dividend news as well as to unexpected part of announcements. In other words, our model is able to explain the state-dependent reaction of volatility to news variables. On non-announcement days, the conditional volatility of unexpected returns depends on investor's prior and current beliefs, the number of periods till the next announcement as well as the dividends news itself. On announcement days, the unexpected part of the announcement also has an effect on how unexpected returns react to dividend news and vice versa. Proposition 7 also shows that the sign of reaction to dividend news or to announcements depends on the news variable itself. In other words, our model is able to generate the asymmetric effect of news on conditional volatility.

Whether it is dividend news or announcements, there are two channels through which additional information affects the conditional volatility of unexpected returns. The first channel is through investor's beliefs about the state variable. The effect of news on the conditional volatility through investor's beliefs depends on the difference between the volatility of the dividend growth process in different states. Any news that increases the probability that the investor assigns to the high growth state increases the conditional volatility if and only if the dividend growth process has higher volatility in high growth state. On the other hand, if the dividend growth process is equally volatile in both states, then this effect of news on conditional volatility disappears. The second channel is through the investor's uncertainty defined as the variance of the investor's beliefs about the state variable, i.e. $\pi_{1,t}(1-\pi_{1,t})$. Any additional information that increases investor's uncertainty about the state variable always increases the conditional volatility of unexpected returns. Which of these two effects dominates depends on investor's beliefs, news variable itself as well as the model parametrization. Under certain parameter restrictions and assumptions, we can characterize the reaction of conditional volatility to dividend news and announcements. The following proposition characterizes the sign of the reaction of conditional volatility to dividend news.

Proposition 8. *(a) Assume that the dividend growth process is more volatile in the low growth state, i.e. $\sigma_{d,2} > \sigma_{d,1}$, and that the investor's assigns a higher probability to the high growth state after observing the current dividend realization or the announcement, i.e. $\pi_{1,t} > \pi_1^* > 0.5$, or equivalently assume that the dividend growth process is more volatile in the high growth state, i.e. $\sigma_{d,2} < \sigma_{d,1}$, and that the investor's assigns a lower probability to the high growth state after observing the current dividend realization or the announcement, i.e. $\pi_{1,t} < \pi_1^* < 0.5$, then the reaction of conditional volatility to dividend news is positive if and only if the dividend realization is large enough, i.e. $\Delta d_t > \Delta d^*$, where π_1^* is a deterministic bound defined as $\pi_1^* = 0.5 - 0.5(\sigma_{d,1}^2 - \sigma_{d,2}^2)/(m_{1,\tau-1} - m_{2,\tau-1})^2$ for $\tau = T_n - t$ and $T_{n-1} < t < T_n$ and $\pi_1^* = 0.5 - 0.5(\sigma_{d,1}^2 - \sigma_{d,2}^2)/(m_{1,T-1} - m_{2,T-1})^2$ for $t = T_n$.*

(b) Assume that the dividend growth process is more volatile in the low growth state, i.e. $\sigma_{d,2} > \sigma_{d,1}$, and that the investor's assigns a lower probability to the high growth state after observing the current dividend realization or the announcement, i.e. $\pi_{1,t} < \pi_1^* < 0.5$, or equivalently assume that the dividend growth process is more volatile in the high growth state, i.e. $\sigma_{d,2} < \sigma_{d,1}$, and that the investor's assigns a higher probability to the high growth state after observing the current dividend realization or the announcement, i.e. $\pi_{1,t} > \pi_1^* > 0.5$, then the reaction of conditional volatility to dividend news is positive if and only if the dividend realization is small enough, i.e. $\Delta d_t < \Delta d^*$.

(c) Assume that the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,2} = \sigma_{d,1}$, then the reaction of conditional volatility to dividend news is positive if and only if the investor's assigns a higher probability to the high growth state after observing the current dividend realization or the announcement, i.e. $\pi_{1,t} > 0.5$.

Proof. All proofs are in the appendix. □

Although Proposition 8 completely characterizes the reaction of conditional volatility to dividend news on announcement and non-announcement periods, we can provide a smaller set of assumptions that characterizes the reaction of conditional volatility to dividend news on non-announcement days. This is due to fact that investor's beliefs on non-announcement days are functions of the dividend realization. Hence, we characterize the regions of the dividend growth process for which the reaction of conditional volatility is unambiguously positive to dividend news in the following corollary.

Corollary 2. *(a) Assume that the volatility of the dividend growth process is different in different states, i.e.* $\sigma_{d,1} \neq \sigma_{d,2}$*, then the reaction of conditional volatility of unexpected returns to dividend news on non-announcement days is positive if the realized dividend is in a certain region of the dividend growth process, i.e.* $\Delta d^* - \delta(\pi_{1,t-1}) < \Delta d_t < \Delta d^*$ *where* $\delta(\pi_{1,t-1})$ *is given in the appendix. Otherwise, the reaction of conditional volatility of unexpected returns to dividend news on non-announcement days can be positive or negative.*

(b) Assume that the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,1} = \sigma_{d,2}$*, then the reaction of conditional volatility to dividend news is positive if the dividend realization is large enough, i.e.* $\Delta d_t > (\mu_{d,1} + \mu_{d,2})/2 - \sigma_{d,1}^2/(\mu_{d,1} - \mu_{d,2}) \log(\pi_{1,t-1}/(1 - \pi_{1,t-1}))$*. Otherwise, the reaction of conditional volatility of unexpected returns to dividend news on non-announcement days can be positive or negative.*

Proof. All proofs are in the appendix. □

Corollary 2 shows that the conditional volatility reacts positively to dividend news on non-announcement periods in certain regions of the dividend growth process. In these regions, the positive effect of dividend news through investor's uncertainty dominates its negative effect through investor's beliefs and the conditional volatility reacts positively to dividend news. However, the reaction can be positive or negative depending on many factors outside these regions. Furthermore, an analog of Corollary 2 is not feasible for announcement days as investor's beliefs on announcement days do not only depend on the dividend realization but also on the announcement. This is also true for the reaction of conditional volatility to the unexpected part of the announcement which we characterize in the following proposition.

Proposition 9. *(a) Assume that the external signal has a higher volatility in the low growth state of the dividend process, i.e.* $\sigma_{x,2} > \sigma_{x,1}$*, and that the investor's assigns a higher probability to the high growth state after observing the current dividend realization and the announcement, i.e.* $\pi_{1,T_n} > \pi_1^* > 0.5$*, or equivalently assume that the external signal has a higher volatility in the high growth state of the dividend process, i.e.* $\sigma_{x,2} < \sigma_{x,1}$*, and that the investor's assigns a higher probability to the low growth state after observing the current dividend realization and the announcement, i.e.* $0.5 < \pi_{1,T_n} < \pi_1^*$*, then the reaction* *of conditional volatility to the announcement is positive if and only if the announcement is large enough, i.e.* $x_n > x^*$*, where* π_1^* *is as defined in Proposition 8 for announcement periods.*

(b) Assume that the external signal has the same volatility in both states of the dividend process, i.e. $\sigma_{x,2} = \sigma_{x,1}$*, then the reaction of conditional volatility to the announcement is positive if and only if the investor's assigns a higher probability to the high growth state after observing the current dividend realization and the announcement, i.e.* $\pi_{1,T_n} > \pi_1^* > 0.5$*, where* π_1^* *is as defined in Proposition 8 for announcement periods and is equal to* 0.5 *if we further assume that the dividend growth process has the same volatility in both states.*

Proof. All proofs are in the appendix. □

To gain intuition about the reaction of conditional volatility to the unexpected part of the announcement characterized in Proposition 9, we focus on the case where we assume both the external signal and the dividend process itself have higher volatilities in the low growth state of the dividend process, i.e. $\sigma_{d,2} > \sigma_{d,1}$ and $\sigma_{x,2} > \sigma_{x,1}$. Under these assumptions, any positive information revealed in the announcement causes the investor to increase the probability that he assigns to the high growth state if and only if the news is good enough, i.e. $x_n > x^*$. This in turn decreases the conditional volatility of unexpected returns, all else equal. However, an increase in the probability that the investor assigns to the high state might also result in an increase in his uncertainty about the state variable. An increase in investor's uncertainty increases the conditional volatility. If the probability of the high growth state is higher than the probability of the low growth state after observing the dividend realization and the announcement, then the effect of uncertainty will dominate the effect of positive news and the conditional volatility would increase. The important benchmark point for the investor's beliefs is when he assigns the same probability to both states of dividend growth process and his uncertainty is almost maximized. Depending on model parameters, news that causes the investor to assign higher probability to the high growth state will result in a increase in the conditional volatility of unexpected returns.

Although the sign of the reaction of conditional volatility to unexpected part of the announcement can be characterized under few assumptions, the same is not true for the magnitude which depends on many factors such as investor's coefficient of risk aversion, the number of periods till the next announcement and investor's prior and current beliefs as well as all other model parameters. Even though we can characterize the magnitude of the reaction of conditional volatility as a function of these factors under several assumptions, here we focus on the effect of the number of periods till the next announcement. The following proposition characterizes the magnitude of reaction of conditional volatility to dividend news as a function of time till the next announcement.

Proposition 10. *(a) Assume that the investor is not a log-utility investor, i.e.* $\gamma \neq 1$ *and that the dividend growth process has a higher volatility in the high growth state, i.e.* $\sigma_{d,2} < \sigma_{d,1}$*, then the magnitude of the reaction of conditional volatility in absolute value in period* t_1 *is greater than that of the reaction in period* t_2 *if and only if* $t_2 > t_1$*, all else equal.*

(b) Assume that the investor is not a log-utility investor, i.e. $\gamma \neq 1$ *and that the dividend growth process has a higher volatility in the low growth state, i.e.* $\sigma_{d,2} > \sigma_{d,1}$*, then the magnitude of the reaction of conditional volatility in absolute value in period* t_1 *is greater than that of the reaction in period* t_2 *if and only if* $t_2 > t_1$ *and* $\pi_{1,t_1}, \pi_{1,t_2} > 0.5$*, all else equal.*

(c) Assume that the investor is a log-utility investor, i.e. $\gamma = 1$*, then the reaction of unexpected returns to dividend news does not depend on the number of periods till the next announcement.*

Proof. All proofs are in the appendix. □

Proposition 10 shows that the sensitivity of conditional volatility to dividend news decreases as the announcement approaches. In other words, the conditional volatility reacts stronger to news released earlier in between announcement periods, every thing else equal. This is again due to the importance of additional information released earlier. The investor puts more weight on information released earlier as he knows that the dividend growth process will be in the same state till the next announcement. Hence, any information about the state variable is more useful for the investor the earlier it is released. Furthermore, not only the sensitivity of conditional volatility but also the conditional volatility changes with respect to the number of periods till the next announcement. The following corollary characterizes the conditional volatility of unexpected returns for a given set of investor's beliefs.

Corollary 3. *(a) Assume that the investor is not a log-utility investor, i.e.* $\gamma \neq 1$ *and that the investor's beliefs in period* t_1 *and* t_2 *are the same, i.e.* $\pi_{1,t_1} = \pi_{1,t_2}$ *where* $T_{n-1} < t_1, t_2 < T_n$ *for* $n = 1, 2, \ldots$*, then the conditional volatility given investor's beliefs at time* t_1 *is greater than the conditional volatility given his beliefs at time* t_2 *if and only if* $t_1 > t_2$*.*

(b) Assume that the investor is not a log-utility investor, i.e. $\gamma \neq 1$ *and that the investor's beliefs a period before the announcement,* $T_n - 1$ *and on the announcement day,* T_n*, after observing the dividend realization and the announcement are the same, i.e.* $\pi_{1,T_n-1} = \pi_{1,T_n}$*, then the conditional volatility at time* T_n *is greater than the conditional volatility at time* $T_n - 1$*, i.e.* $var_{T_n}(r_{T_n+1}) > var_{T_n-1}(r_{T_n})$*.*

Part (a) of Corollary 3 shows that the conditional volatility of unexpected returns decreases for a given set of investor's beliefs as the announcement approaches. Another interpretation of part (a) is that the unconditional level of volatility decreases as the announcement approaches. This is again due to the higher importance of information revealed earlier. Part (b) shows that the conditional volatility is higher on an announcement period than it is one period before the announcement. In other words, the conditional volatility of unexpected returns decreases as the announcement approaches and it jumps to a higher level on the announcement day. Corollary 3 shows that our model is capable of generating another empirical fact that the conditional volatility of returns decreases significantly the day before an announcement, dubbed the "calm-before-the-storm" by Jones, Lamont, and Lumsdaine (1998).

4 Conclusion

This paper analyzes how dynamics of stock returns are affected by news about the state of the economy in a Lucas-type model where investors never observe the true growth rate of the economy but rather infer about it through two different sources of information. In between announcement periods, investors observe dividend realizations and update their beliefs about the current state of the economy. On announcement periods, investors receive an additional external signal about the state of the economy. In this framework, we characterize the reaction of both level and conditional volatility of returns to unexpected information from these two separate sources. Our model is able to account for many of the recent empirical findings on the reaction of stock returns to macroeconomic announcements.

First of all, our model is able to generate time-varying conditional expected returns, conditional volatility as well as volatility clustering. The conditional volatility of unexpected returns can be expressed as an ARCH-type model where the conditional volatility depends on lagged conditional volatility, the news variables as well as the change in investor's uncertainty. We distinguish between dividend news and unexpected part of announcements and show that our model can also account for time- and state-dependent reaction of returns to these two sources of additional information.

Secondly, we show that the reaction to these two news variable are quite different as dividend news affects not only investor's beliefs about the state of the economy but also his current consumption whereas external signals only affect his beliefs. Under a certain set of assumptions, we show that the reaction of stock returns to dividend news is positive whereas the reaction to an external signal is negative under the same set of assumptions. The reaction of returns to dividend news is also a function of the number of periods till the next announcement. All else equal, we find that a news variable has a greater impact on investor's beliefs the earlier it is released. For two dividend news of the same magnitude, the reaction to the one released earlier is stronger than to the one released later assuming that the reaction to both news is positive.

Third, we show that the reaction of conditional volatility depends on the magnitude of the news variable itself which suggests that our model can account for the asymmetric news effect on conditional volatility. We then show that dividend news and external signal affect conditional volatility in a similar fashion in contrast to their effect on the level of returns. Under certain assumptions, the conditional volatility reacts positively to a news variable. For example, assuming that the dividend growth process (the external signal) is more volatile in the low growth state and that the investor assigns a higher probability to the high growth state after observing the dividend realization, the reaction of conditional volatility is positive if and only if the dividend realization (the external signal) is large enough. All else equal, the conditional volatility and its sensitivity to dividend news decrease as the announcement approaches which can account for the "calm-before-the-storm" effect.

Although our model is realistic, analytically tractable and most importantly suitable for the question addressed in this paper, it has shortcomings like any other model. First of all, to obtain analytical solutions we assume that the investor has a power utility. In this framework, investor's relative degree of risk aversion and his intertemporal elasticity of substitution is closely linked making it impossible to distinguish their effects on returns. One can extend the model where the investor has an Epstein-Zin type utility (Epstein and Zin (1989)) at the cost of losing analytical solutions. However, it is still possible to obtain analytical solutions for several extensions of our model. For example, one can think of modeling consumption and dividend processes separately as in Cecchetti, Lam, and Mark (1993) to analyze the reaction of stock returns in a partial equilibrium framework. Another possible extension of our model where analytical solutions might be still feasible is to model dividends and the price of the consumption good separately as in David and Veronesi (2004). In this framework, one can consider analyzing the effect of releases about interest rates.

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A Proofs

Proof of Lemma 1. Investor's beliefs need to be characterized for three different time periods. Case 1 is the investor's beliefs on the previous announcement period, $t = T_{n-1}$, Case 2 is the investor's beliefs in between announcement days, $T_{n-1} < t < T_n$, and finally Case 3 is the investor's beliefs on the current announcement period, $t = T_n$. We analyze these three cases separately.

Case 1. ($t = T_{n-1}$): On the previous announcement day, the investor knows that the dividend growth process might have possibly switched to a new state. He updates his beliefs based on his beliefs about the previous state variable, $\Pr(S_{n-1} = i|\mathcal{F}_{T_{n-1}})$, and the law of motion for the state variable, i.e. the transition probability matrix, $\mathbf{Q}$. Hence, the investor's beliefs about the current state variable is a weighted average of transition probabilities where the weights are the investor's beliefs about the previous state variable.

Case 2. ($T_{n-1} < t < T_n$): In between announcement days, the only source of information about the state variable is the dividend realization. The investor updates his beliefs from the previous period according to the Bayes' rule based on the observed dividend realization. Recall that the probability of being in state j, $\pi_{j,t} = \Pr(S_n = j|\mathcal{F}_t)$.

$$\pi_{j,t} = \Pr(S_n = j|\Delta d_t, \mathcal{F}_{t-1}) \tag{23}$$

$$= \frac{\Pr(\Delta d_t|S_n = j, \mathcal{F}_{t-1}) \Pr(S_n = j|\mathcal{F}_{t-1})}{\Pr(\Delta d_t|\mathcal{F}_{t-1})} \tag{24}$$

$$= \frac{\Pr(\Delta d_t|S_n = j, \mathcal{F}_{t-1}) \Pr(S_n = j|\mathcal{F}_{t-1})}{\sum_{i=1}^{N} \Pr(\Delta d_t|S_n = i, \mathcal{F}_{t-1}) \Pr(S_n = i|\mathcal{F}_{t-1})} \tag{25}$$

$$= \frac{\phi(\frac{\Delta d_t - \mu_{d,j}}{\sigma_{d,j}})\pi_{j,t-1}}{\sum_{i=1}^{N} \phi(\frac{\Delta d_t - \mu_{d,i}}{\sigma_{d,i}})\pi_{i,t-1}} \tag{26}$$

where $\phi(\cdot)$ is the standard normal density function. Equations (23) follows from the definition of the information set, $\mathcal{F}_t$, which includes all information and the current dividend realization. Equation (24) and (25) follow from Bayes' rule and law of total probability, respectively.[7] Note that, by definition, $\pi_{i,t-1} = \Pr(S_n = i|\mathcal{F}_{t-1})$. Equation (26) follows from the law of motion for dividend growth in Equation (2).

Case 3. ($t = T_n$): On the announcement day, T_n, there are two sources of information about the state variable, the dividend realization and the announcement. The investor updates his beliefs from the previous

[7] Recall that Bayes' rule is $\Pr(A|B,C) = \frac{\Pr(B|A,C)\Pr(A|C)}{\Pr(A|C)}$

period according to the Bayes' rule based on the observed dividend realization and the announcement.

$$\pi_{j,T_n} = \Pr(S_n = j|\Delta d_{T_n}, x_n, \mathcal{F}_{T_n-1}) \tag{27}$$
$$= \frac{\Pr(\Delta d_{T_n}|S_n = j, \mathcal{F}_{T_n-1}) \Pr(x_n|S_n = j, \mathcal{F}_{T_n-1}) \Pr(S_n = j|\mathcal{F}_{T_n-1})}{\Pr(\Delta d_t, x_n|\mathcal{F}_{T_n-1})} \tag{28}$$
$$= \frac{\Pr(\Delta d_{T_n}|S_n = j, \mathcal{F}_{T_n-1}) \Pr(x_n|S_n = j, \mathcal{F}_{T_n-1}) \Pr(S_n = j|\mathcal{F}_{T_n-1})}{\sum_{i=1}^{N} \Pr(\Delta d_{T_n}|S_n = i, \mathcal{F}_{T_n-1}) \Pr(x_n|S_n = i, \mathcal{F}_{T_n-1}) \Pr(S_n = i|\mathcal{F}_{T_n-1})} \tag{29}$$
$$= \frac{\phi(\frac{\Delta d_{T_n}-\mu_{d,j}}{\sigma_{d,j}})\phi(\frac{x_n-\mu_{x,j}}{\sigma_{x,j}})\pi_{j,T_n-1}}{\sum_{i=1}^{N} \phi(\frac{\Delta d_{T_n}-\mu_{d,i}}{\sigma_{d,i}})\phi(\frac{x_n-\mu_{x,i}}{\sigma_{x,i}})\pi_{i,T_n-1}} \tag{30}$$

The proof of Case 3 is similar to that of Case 2. Equation (27) follows from the definition of the information set on the announcement day T_n, $\mathcal{F}_{T_n}$, which includes all past information, the current dividend realization and the announcement. Equations (28) and (29) follow from the independence of Δd_{T_n} and x_n conditional on the current state variable. Equation (30) follows from the law of motion for dividend growth in Equation (2) and the law of motion for external public signal in Equation (3).

□

Proof of Lemma 2. By recursive substitution of future prices into Euler equation in (8), the price of the risky asset can be expressed as a discounted sum of expected future dividends where the discount factor is the intertemporal marginal rate of substitution:

$$P_t = E_t\Big[\sum_{\tau=1}^{\infty} \beta^{\tau} \frac{U'(C_{t+\tau})}{U'(C_t)} D_{t+\tau}\Big] \tag{31}$$

Imposing the equilibrium condition, $C_t = D_t$, substituting the functional form for the utility function and rearranging the terms, the price-dividend ratio at time t can be expressed as follows:

$$\frac{P_t}{D_t} = E_t\Big[\sum_{\tau=1}^{\infty} \beta^{\tau} \Big(\frac{D_{t+\tau}}{D_t}\Big)^{1-\gamma}\Big] \tag{32}$$

The infinite sum in Equation (32) can be expressed as a sum of two terms, sum of discounted future dividends until the upcoming announcement day and sum of discounted future dividends after the upcoming announcement day. The price-dividend ratio can be expressed as follows:

$$\frac{P_t}{D_t} = \sum_{\tau=1}^{T_n-t} \beta^{\tau} E_t\Big[\Big(\frac{D_{t+\tau}}{D_t}\Big)^{1-\gamma}\Big] + \beta^{T_n-t} E_t\Big[\Big(\frac{D_{T_n}}{D_t}\Big)^{1-\gamma} \frac{P_{T_n}}{D_{T_n}}\Big] \tag{33}$$

Conditioning on the current state, the following holds:

$$\frac{P_t}{D_t} = \sum_{i=1}^{N} \sum_{\tau=1}^{T_n-t} \beta^{\tau} E_t\Big[\Big(\frac{D_{t+\tau}}{D_t}\Big)^{1-\gamma}\Big| S_n = i\Big]\pi_{it}$$
$$+ \sum_{i=1}^{N} \beta^{T_n-t} E_t\Big[\Big(\frac{D_{T_n}}{D_t}\Big)^{1-\gamma}\Big| S_n = i\Big] E_t\Big[\frac{P_{T_n}}{D_{T_n}}\Big| S_n = i\Big]\pi_{it} \tag{34}$$

where Equation (34) follows from law of total probability and conditional independence of $\frac{D_{T_n}}{D_t}$ and $\frac{P_{T_n}}{D_{T_n}}$ when the conditioning information is the current state variable. Note that for t such that $T_{n-1} \leq t \leq T_n$ and $1 \leq \tau \leq T_n - t$, we have

$$E_t\left[\left(\frac{D_{t+\tau}}{D_t}\right)^{1-\gamma}\middle|S_n = i\right] = E_t[\exp((1-\gamma)\mu_{d,i}\tau + (1-\gamma)\sigma_{d,i}\sum_{l=1}^{\tau}\varepsilon_{t+l})] \tag{35}$$

$$= \exp((1-\gamma)\mu_{d,i} + (1-\gamma)^2\sigma_{d,i}^2/2)^{\tau} \tag{36}$$

$$\equiv (e^{a_i})^{\tau} \tag{37}$$

where $a_i \equiv (1-\gamma)\mu_{d,i} + (1-\gamma)^2\sigma_{d,i}^2/2$. Equation (35) follows from the law of motion for the dividend growth rate. Equation (36) follows from the formula for the expectation of a lognormal variable where the mean and variance of the normal variable are $(1-\gamma)\mu_{d,i}\tau$ and $(1-\gamma)^2\sigma_{d,i}^2\tau$, respectively. The price-dividend ratio can be expressed as:

$$\frac{P_t}{D_t} = \sum_{i=1}^{N}\sum_{\tau=1}^{T_n-t}(\beta e^{a_i})^{\tau}\pi_{it} + \sum_{i=1}^{N}(\beta e^{a_i})^{T_n-t}E_t\left[\frac{P_{T_n}}{D_{T_n}}\middle|S_n = i\right]\pi_{it}$$

$$= \sum_{i=1}^{N}\left(\frac{(\beta e^{a_i})^{T_n-t+1}-1}{\beta e^{a_i}-1} - 1\right)\pi_{it} + \sum_{i=1}^{N}(\beta e^{a_i})^{T_n-t}E_t\left[\frac{P_{T_n}}{D_{T_n}}\middle|S_n = i\right]\pi_{it} \tag{38}$$

The price-dividend ratio on the previous announcement day T_{n-1} can be expressed as follows by setting $t = T_{n-1}$:

$$\frac{P_{T_{n-1}}}{D_{T_{n-1}}} = \sum_{i=1}^{N}\left(\sum_{j=1}^{N}\left(\frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1} - 1\right)q_{i,j} + \sum_{j=1}^{N}(\beta e^{a_j})^{T}E_t\left[\frac{P_{T_n}}{D_{T_n}}\middle|S_n = j\right]q_{i,j}\right)\pi_{i,T_{n-1}} \tag{39}$$

where $q_{i,j}$ is the ij^{th} element of the transition probability matrix $\mathbf{Q}$.

In order to solve the difference equation in (39), we conjecture a solution for the price-dividend ratio on announcement periods of the following form:

$$\frac{P_{T_n}}{D_{T_n}} = \lambda_i \text{ for } n = 1, 2, \ldots \text{ and } i = 1, 2, \ldots, N \tag{40}$$

Plugging in the conjecture in Equation (40), we obtain the following system of N linear equations in N variables, $(\lambda_1, \ldots, \lambda_N)$:

$$\lambda_i = \sum_{j=1}^{N}\left(\frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1} - 1\right)q_{ij} + \left(\sum_{j=1}^{N}(\beta e^{a_j})^{T}q_{ij}\right)\left(\sum_{j=1}^{N}\lambda_j q_{ij}\right) \tag{41}$$

for $i = 1, 2, \ldots, N$. To reduce notation, we define a $N \times 1$ vector, $\mathbf{G}$, whose j^{th} element, g_j, is given by $g_j = \frac{(\beta e^{a_j})^{T+1}-1}{\beta e^{a_j}-1} - 1$ and a $N \times N$ diagonal matrix, $\mathbf{H}$, whose i^{th} diagonal element, h_i, is given by $h_i = \sum_{j=1}^{N}(\beta e^{a_j})^{T}q_{ij}$. The system of equations in (41) can be expressed as follows:

$$\lambda = \mathbf{QG} + \mathbf{HQ}\lambda \tag{42}$$

Solving for the vector λ, we obtain the price-dividend ratio on announcement days in Lemma 2.

First note that elements of $\mathbf{HQ}$ and $\mathbf{QG}$ are non-negative. To prove that elements of λ are non-negative, it suffices to show that the elements of $(\mathbf{I}-\mathbf{HQ})^{-1}$ are non-negative. According to Theorem III* of Debreu and Herstein (1953), the elements of $(\mathbf{I}-\mathbf{HQ})^{-1}$ are non-negative if and only if the maximal non-negative characteristic root of $\mathbf{HQ}$ is less than 1. Let p denote the maximal nonnegative characteristic root of $\mathbf{HQ}$. According to the Perron-Frobenius theorem for non-negative matrices, we know that $\min_i \sum_{j=1}^{N}[\mathbf{HQ}]_{i,j} \leq p \leq \max_i \sum_{j=1}^{N}[\mathbf{HQ}]_{i,j}$ where $[\cdot]_{i,j}$ refers to the ij^{th} element of the matrix. We know that $\sum_{j=1}^{N}[\mathbf{HQ}]_{i,j} = h_i < 1$ and thus $p < 1$. This completes the proof.

□

Proof of Proposition 1. Proof of Proposition follows from Equation (38). Note that $E_t[\frac{P_{T_n}}{D_{T_n}}|S_n = j] = \lambda_j$ from the result in Lemma 2. Plugging in, we obtain Equation (9) for the price-dividend ratio on non-announcement days. □

Proof of Proposition 2. Using a first-order Taylor expansion of the log function around the long term average of the dividend price ratio, log returns on the risky asset can be expressed as follows:

$$\begin{aligned} r_t &= \log(1+P_t/D_t) - \log(P_{t-1}/D_{t-1}) + \Delta d_t \\ &\approx \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(P_t/D_t - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \Delta d_t \end{aligned} \tag{43}$$

Using the above approximation, the conditional expectation of log returns based on the information set at time $t-1$ can be written as follows:

$$E_{t-1}[r_t] = \log(1+\overline{\lambda}) + \frac{1}{1+\overline{\lambda}}(\sum_{j=1}^{N} k_{j,\tau}\pi_{j,t-1} - \overline{\lambda}) - \log(\overline{\lambda}) - \frac{1}{\overline{\lambda}}(P_{t-1}/D_{t-1} - \overline{\lambda}) + \sum_{j=1}^{N} \mu_{d,j}\pi_{j,t-1} \tag{44}$$

The unexpected log return on the risky asset in Equation (11) can be obtained as the difference between Equations (43) and (44). The long term average of the dividend price ratio is the unconditional expectation of the price-dividend ratio as in Proposition 2.

For notational simplicity, let $n_{i,\tau}$ denote $k_{i,\tau} + \mu_{d,i}$, then the conditional volatility of unexpected returns can be written as follows:

$$
\begin{aligned}
\text{var}_t(r_{t+1} - E_t[r_{t+1}]) &= \text{var}_t\Big(\sum_{i=1}^{N} n_{i,\tau-1}\pi_{i,t+1} + \Delta d_{t+1}\Big) \\
&= E_t\Big[\Big(\sum_{i=1}^{N} n_{i,\tau-1}\pi_{i,t+1} + \Delta d_{t+1}\Big)^2\Big] - \Big(E_t\Big[\sum_{i=1}^{N} n_{i,\tau-1}\pi_{i,t+1} + \Delta d_{t+1}\Big]\Big)^2 \\
&= E_t\Big[\sum_{i=1}^{N} n_{i,\tau-1}^2\pi_{i,t+1}^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} n_{i,\tau-1}n_{j,\tau-1}\pi_{i,t+1}\pi_{j,t+1} \\
&+ 2\sum_{i=1}^{N} n_{i,\tau-1}\pi_{i,t+1}\Delta d_{t+1} + \Delta d_{t+1}^2\Big] - \Big(\sum_{i=1}^{N}(n_{i,\tau-1} + \mu_{d,i})\pi_{i,t}\Big)^2 \\
&= \sum_{i=1}^{N}\Big(n_{i,\tau-1}^2 + \mu_{d,i}^2 + \sigma_{d,i}^2 + 2n_{i,\tau-1}\mu_{d,i}\Big)\pi_{i,t} - \Big(\sum_{i=1}^{N}(n_{i,\tau-1} + \mu_{d,i})\pi_{i,t}\Big)^2 \\
&= \sum_{i=1}^{N}(n_{i,\tau-1} + \mu_{d,i})^2\pi_{i,t} + \sum_{i=1}^{N}\sigma_{d,i}^2\pi_{i,t} - \Big(\sum_{i=1}^{N}(n_{i,\tau-1} + \mu_{d,i})\pi_{i,t}\Big)^2
\end{aligned}
$$

Plugging in the definition of $n_{i,\tau}$, we obtain the conditional volatility of unexpected returns in Proposition 2. These results are based on the following relations: $E_{t-1}[\pi_{i,t}] = E_{t-1}[\pi_{i,t}^2] = \pi_{i,t-1}$, $E_{t-1}[\pi_{i,t}\Delta d_t] = \mu_{d,i}\pi_{i,t-1}$, $E_{t-1}[\pi_{i,t}\pi_{j,t}] = 0$ for $i \neq j$. □

Proof of Corollary 1. Equation (13) can be directly obtained from Equation (11) by setting $N = 2$. The conditional volatility of unexpected returns can be expressed as an ARCH-type as the difference between $\text{var}_t(r_{t+1})$ and $(m_{1,\tau-1} - m_{2,\tau-1})^2/(m_{1,\tau} - m_{2,\tau})^2\text{var}_{t-1}(r_t)$ for $N = 2$. □

Proof of Proposition 3. Recall that $\pi_{1,t}$ denotes the probability that the investor assigns to the high growth state when there are only two possible states of the dividend growth process. For a non-announcement period t such that $T_{n-1} < t < T_n$, $\pi_{1,t}$ can be expressed as follows:

$$
\pi_{1,t} = \Bigg[1 + \frac{1-\pi_{1,t-1}}{\pi_{1,t-1}}\exp\Big(-\frac{(\mu_{d,1}-\mu_{d,2})^2}{2(\sigma_{d,2}^2-\sigma_{d,1}^2)}\Big)\exp\Big(\frac{\sigma_{d,2}^2-\sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}\Big(u_{d,t} - \frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}_{d,t-1}^2}{\sigma_{d,2}^2-\sigma_{d,1}^2}\Big)^2\Big)\Bigg]^{-1}
$$

and for an announcement period, T_n, π_{1,T_n} can be expressed as follows:

$$
\begin{aligned}
\pi_{1,T_n} = \Bigg[&1 + \frac{1-\pi_{1,T_n-1}}{\pi_{1,T_n-1}}\exp\Big(-\frac{(\mu_{d,1}-\mu_{d,2})^2}{2(\sigma_{d,2}^2-\sigma_{d,1}^2)}\Big)\exp\Big(-\frac{(\mu_{x,1}-\mu_{x,2})^2}{2(\sigma_{x,2}^2-\sigma_{x,1}^2)}\Big) \\
&\exp\Big(\frac{\sigma_{d,2}^2-\sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}\Big(u_{d,t} - \frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}_{d,t-1}^2}{\sigma_{d,2}^2-\sigma_{d,1}^2}\Big)^2\Big) \\
&\exp\Big(\frac{\sigma_{x,2}^2-\sigma_{x,1}^2}{2\sigma_{x,1}^2\sigma_{x,2}^2}\Big(u_{x,t} - \frac{(\mu_{x,1}-\mu_{x,2})\overline{\sigma}_{x,t-1}^2}{\sigma_{x,2}^2-\sigma_{x,1}^2}\Big)^2\Big)\Bigg]^{-1}
\end{aligned}
$$

Then the derivative of $\pi_{1,t}$ on a non-announcement period t such that $T_{n-1} < t < T_n$ is given by

$$
\partial\pi_{1,t}/\partial u_{d,t} = f_1(u_{d,t}, \pi_{1,t-1})\Big(\frac{\sigma_{d,2}^2-\sigma_{d,1}^2}{\sigma_{d,1}^2\sigma_{d,2}^2}\Big)\Big(u_{d,t} - \frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}_{d,t-1}^2}{\sigma_{d,2}^2-\sigma_{d,1}^2}\Big)
$$

where

$$f_1(u_{d,t}, \pi_{1,t-1}) = \kappa_1(u_{d,t}, \pi_{1,t-1})/(1 + \kappa_1(u_{d,t}, \pi_{1,t-1}))^2$$

and

$$\kappa_1(u_{d,t}, \pi_{1,t-1}) = \frac{1 - \pi_{1,t-1}}{\pi_{1,t-1}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma^2_{d,2} - \sigma^2_{d,1})}\right) \exp\left(\frac{\sigma^2_{d,2} - \sigma^2_{d,1}}{2\sigma^2_{d,1}\sigma^2_{d,2}}\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2} - \sigma^2_{d,1}}\right)^2\right)$$

On the other hand, the derivative of π_{1,T_n} with respect to $\partial u_{d,T_n}$ on an announcement period T_n is given by

$$\partial \pi_{1,T_n}/\partial u_{d,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\left(\frac{\sigma^2_{d,2} - \sigma^2_{d,1}}{\sigma^2_{d,1}\sigma^2_{d,2}}\right)\left(u_{d,T_n} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}^2_{d,T_n-1}}{\sigma^2_{d,2} - \sigma^2_{d,1}}\right)$$

and the derivative of π_{1,T_n} with respect to $\partial u_{x,T_n}$ can be expressed in a similar fashion as follows:

$$\partial \pi_{1,T_n}/\partial u_{x,T_n} = f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})\left(\frac{\sigma^2_{x,2} - \sigma^2_{x,1}}{\sigma^2_{x,1}\sigma^2_{x,2}}\right)\left(u_{x,T_n} - \frac{(\mu_{x,1} - \mu_{x,2})\overline{\sigma}^2_{x,T_n-1}}{\sigma^2_{x,2} - \sigma^2_{x,1}}\right)$$

where

$$f_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1}) = \kappa_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1})/(1 + \kappa_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1}))^2$$

and

$$\begin{aligned}\kappa_2(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1}) &= \frac{1 - \pi_{1,T_n-1}}{\pi_{1,T_n-1}} \exp\left(-\frac{(\mu_{d,1} - \mu_{d,2})^2}{2(\sigma^2_{d,2} - \sigma^2_{d,1})}\right) \exp\left(-\frac{(\mu_{x,1} - \mu_{x,2})^2}{2(\sigma^2_{x,2} - \sigma^2_{x,1})}\right) \\ &\quad \exp\left(\frac{\sigma^2_{d,2} - \sigma^2_{d,1}}{2\sigma^2_{d,1}\sigma^2_{d,2}}\left(u_{d,t} - \frac{(\mu_{d,1} - \mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2} - \sigma^2_{d,1}}\right)^2\right) \\ &\quad \exp\left(\frac{\sigma^2_{x,2} - \sigma^2_{x,1}}{2\sigma^2_{x,1}\sigma^2_{x,2}}\left(u_{x,t} - \frac{(\mu_{x,1} - \mu_{x,2})\overline{\sigma}^2_{x,t-1}}{\sigma^2_{x,2} - \sigma^2_{x,1}}\right)^2\right)\end{aligned}$$

It is easy to see that both f_1 and f_2 are positive functions ad bounded above by 0.25. The derivative of unexpected returns with respect to dividend news, $\partial r^*_t/\partial u_{d,t}$, is given by

$$\partial r^*_t/\partial u_{d,t} = 1 + \frac{(k_{1,\tau} - k_{2,\tau})}{1 + \overline{\lambda}}\partial \pi_{1,t}/\partial u_{d,t}$$

where $\tau = T_n - t$. Equations (17) and (18) of Proposition 3 can be obtained by plugging in the appropriate derivative of $\pi_{1,t}$ with respect to dividend news in the above equation. The derivative of unexpected returns with respect to the unexpected part of the announcement, $\partial r^*_{T_n}/\partial u_{x,T_n}$, is given by

$$\partial r^*_{T_n}/\partial u_{x,T_n} = \frac{(\lambda_1 - \lambda_2)}{1 + \overline{\lambda}}\partial \pi_{1,T_n}/\partial u_{x,T_n}$$

and Equation (19) of Proposition 3 can be obtained by plugging in the derivative of π_{1,T_n} with respect to the unexpected part of the announcement. □

Proof of Proposition 4. To prove Proposition 4, we start by showing that λ_j is a non-increasing function of μ_i for $i, j = 1, 2, \ldots, N$ if and only if the investor is more risk averse than a log utility investor. In other words, $\partial\lambda_j/\partial\mu_i$ is non-positive for $i, j = 1, 2, \ldots, N$ if and only if $\gamma > 1$. To show this, first note that $\partial a_j/\partial\mu_i$ is zero for $i \neq j$ and negative for $i = j$ if and only if $\gamma > 1$. This implies that the diagonal elements of $\partial\mathbf{H}/\partial\mu_i$ and all elements of $\partial\mathbf{G}/\partial\mu_i$ are negative if and only if $\gamma > 1$. This in turn implies

$$\begin{aligned}
\frac{\partial\lambda}{\partial\mu_i} &= \frac{\partial(\mathbf{I}-\mathbf{HQ})^{-1}\mathbf{QG}}{\partial\mu_i} \\
&= \frac{\partial(\mathbf{I}-\mathbf{HQ})^{-1}}{\partial\mu_i}\mathbf{QG} + (\mathbf{I}-\mathbf{HQ})^{-1}\mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_i} \\
&= -(\mathbf{I}-\mathbf{HQ})^{-1}\frac{\partial(\mathbf{I}-\mathbf{HQ})}{\partial\mu_i}(\mathbf{I}-\mathbf{HQ})^{-1}\mathbf{QG} + (\mathbf{I}-\mathbf{HQ})^{-1}\mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_i} \\
&= (\mathbf{I}-\mathbf{HQ})^{-1}\left[\frac{\partial\mathbf{H}}{\partial\mu_i}\mathbf{Q}\lambda + \mathbf{Q}\frac{\partial\mathbf{G}}{\partial\mu_i}\right]
\end{aligned}$$

is non-positive if and only if $\gamma > 1$ since we know that elements of $(\mathbf{I}-\mathbf{HQ})^{-1}$ are nonnegative. This implies that $\lambda_1 \leq \lambda_2$, $k_{1,\tau} \leq k_{2,\tau}$ and $m_{1,\tau} \leq m_{2,\tau}$ for $\tau = 0, 1, \ldots, T-1$ if and only if $\gamma > 1$. Further, note that

$$\begin{aligned}
u_{d,t} - \frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2}-\sigma^2_{d,1}} &= \Delta d_t - \frac{\sigma^2_{d,2}\mu_{d,1}-\sigma^2_{d,1}\mu_{d,2}}{\sigma^2_{d,2}-\sigma^2_{d,1}} = \Delta d_t - \Delta d^* \\
u_{x,t} - \frac{(\mu_{x,1}-\mu_{x,2})\overline{\sigma}^2_{x,t-1}}{\sigma^2_{x,2}-\sigma^2_{x,1}} &= x_n - \frac{\sigma^2_{x,2}\mu_{x,1}-\sigma^2_{x,1}\mu_{x,2}}{\sigma^2_{x,2}-\sigma^2_{x,1}} = x_n - x^*
\end{aligned}$$

Proofs of (a) and (b). Here, we only prove the first case of Part (a) of Proposition 4, the proofs of other cases of Part (a) and (b) are similar, hence omitted. Under the assumptions of the first case of Part (a), the indirect effect of dividend news on non-announcement periods

$$f_1(u_{d,t},\pi_{1,t-1})\left(\frac{k_{2,\tau}-k_{1,\tau}}{1+\overline{\lambda}}\right)\left(\frac{\sigma^2_{d,2}-\sigma^2_{d,1}}{\sigma^2_{d,1}\sigma^2_{d,2}}\right)\left(u_{d,t}-\frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2}-\sigma^2_{d,1}}\right)$$

or on announcement periods

$$f_2(u_{d,t},u_{x,t},\pi_{1,t-1})\left(\frac{k_{2,\tau}-k_{1,\tau}}{1+\overline{\lambda}}\right)\left(\frac{\sigma^2_{d,2}-\sigma^2_{d,1}}{\sigma^2_{d,1}\sigma^2_{d,2}}\right)\left(u_{d,t}-\frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}^2_{d,t-1}}{\sigma^2_{d,2}-\sigma^2_{d,1}}\right)$$

are positive if and only if $\Delta d_t - \Delta d^*$ is positive, i.e. $\Delta d_t > \Delta d^*$.

Proof of (c). If the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,2} = \sigma_{d,1}$, then the derivative of unexpected returns to dividend news on non-announcement periods can be expressed as follows:

$$\partial r^*_t/\partial u_{d,t} = 1 + f_3(u_{d,t},\pi_{1,t-1})\left(\frac{\mu_{d,1}-\mu_{d,2}}{\sigma^2_{d,1}}\right)\left(\frac{k_{1,\tau}-k_{2,\tau}}{1+\overline{\lambda}}\right)$$

where

$$f_3(u_{d,t},\pi_{1,t-1}) = \frac{\kappa_3(u_{d,t},\pi_{1,t-1})}{(1+\kappa_3(u_{d,t},\pi_{1,t-1}))^2}$$

and

$$\kappa_3(u_{d,t},\pi_{1,t-1}) = \frac{1-\pi_{1,t-1}}{\pi_{1,t-1}} \exp\left(\frac{2(\mu_{d,2}-\mu_{d,1})\overline{\mu}_{d,t-1}+\mu_{d,1}^2-\mu_{d,2}^2}{2\sigma_{d,1}^2}\right)\exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_{d,1}^2}u_{d,t}\right)$$

On announcement periods, the derivative of unexpected returns to dividend news can be expressed as follows:

$$\partial r_{T_n}^*/\partial u_{d,T_n} = 1 + f_4(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1})\left(\frac{\mu_{d,1}-\mu_{d,2}}{\sigma_{d,1}^2}\right)\left(\frac{\lambda_1-\lambda_2}{1+\overline{\lambda}}\right)$$

where

$$f_4(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1}) = \frac{\kappa_4(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1})}{(1+\kappa_4(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1}))^2}$$

and

$$\begin{aligned}\kappa_4(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1}) \quad &= \quad \frac{1-\pi_{1,T_n-1}}{\pi_{1,T_n-1}}\exp\left(\frac{2(\mu_{d,2}-\mu_{d,1})\overline{\mu}_{d,T_n-1}+\mu_{d,1}^2-\mu_{d,2}^2}{2\sigma_{d,1}^2}\right)\\ &\quad \exp\left(\frac{\mu_{d,2}-\mu_{d,1}}{\sigma_{d,1}^2}u_{d,T_n}\right)\exp\left(\frac{\sigma_{x,2}^2-\sigma_{x,1}^2}{2\sigma_{x,1}^2\sigma_{x,2}^2}\left(u_{x,T_n}-\frac{(\mu_{x,1}-\mu_{x,2})\overline{\sigma}_{x,T_n-1}^2}{\sigma_{x,2}^2-\sigma_{x,1}^2}\right)^2\right)\end{aligned}$$

We should note that f_3 and f_4 are also positive functions and bounded above by 0.25. The reaction of unexpected returns to dividend news on announcement and non-announcement periods is unambiguously positive if the condition in Proposition 4 is satisfied since f_3 and f_4 are positive and bounded above by 0.25.
Proof of (d). The reaction of unexpected returns to dividend news is always positive when $\gamma < 1$ since $k_{1,\tau} > k_{2,\tau}$ if $\gamma < 1$.
Proof of (e). The derivative of unexpected returns with respect to dividend news is always positive and equal to one when $\gamma = 1$ since $k_{1,\tau} = k_{2,\tau}$ if $\gamma = 1$. □

Proof of Proposition 5. Proof of (a) The proof of Part (a) of Proposition 5 is similar to that of Proposition 4. Under the assumptions of the first case of Part (a), the effect of the unexpected part of the announcement

$$f_2(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1})\left(\frac{\lambda_2-\lambda_1}{1+\overline{\lambda}}\right)\left(\frac{\sigma_{x,2}^2-\sigma_{x,1}^2}{\sigma_{x,1}^2\sigma_{x,2}^2}\right)\left(u_{x,T_n}-\frac{(\mu_{x,1}-\mu_{x,2})\overline{\sigma}_{x,T_n-1}^2}{\sigma_{x,2}^2-\sigma_{x,1}^2}\right)$$

is positive if and only if $x_n - x^*$ is positive, i.e. $x_n > x^*$.
Proof of (b) and (c) If the external signal has the same volatility in both states, i.e. $\sigma_{x,2} = \sigma_{x,1}$, then the derivative of unexpected returns to the unexpected part of the announcement can be written as follows:

$$\partial r_{T_n}^*/\partial u_{x,T_n} = f_5(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1})\left(\frac{\mu_{x,1}-\mu_{x,2}}{\sigma_{x,1}^2}\right)\left(\frac{\lambda_1-\lambda_2}{1+\overline{\lambda}}\right)$$

where

$$f_5(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1}) = \frac{\kappa_5(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1})}{(1+\kappa_5(u_{d,T_n},u_{x,T_n},\pi_{1,T_n-1}))^2}$$

and

$$\kappa_5(u_{d,T_n}, u_{x,T_n}, \pi_{1,T_n-1}) = \frac{1-\pi_{1,T_n-1}}{\pi_{1,T_n-1}} \exp\left(\frac{2(\mu_{x,2}-\mu_{x,1})\overline{\mu}_{x,T_n-1}+\mu_{x,1}^2-\mu_{x,2}^2}{2\sigma_{x,1}^2}\right)$$
$$\exp\left(\frac{\mu_{x,2}-\mu_{x,1}}{\sigma_{x,1}^2}u_{x,T_n}\right)\exp\left(\frac{\sigma_{d,2}^2-\sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}\left(u_{d,T_n}-\frac{(\mu_{d,1}-\mu_{d,2})\overline{\sigma}_{d,T_n-1}^2}{\sigma_{d,2}^2-\sigma_{d,1}^2}\right)^2\right)$$

We should note that f_5 is also a positive function and bounded above by 0.25. The reaction of unexpected returns to the unexpected part of the announcement is negative if and only if $\gamma > 1$ since $\lambda_1 < \lambda_2$ if and only if $\gamma > 1$.

Proof of (e). The reaction of unexpected returns to dividend news is independent of investor's beliefs if and only if $\gamma = 1$ since $\lambda_1 = \lambda_2$ if and only if $\gamma = 1$. □

Proof of Proposition 6. To prove Proposition 6, we start by showing that $k_{1,\tau} < k_{1,\tau-1}$ and $k_{2,\tau} > k_{2,\tau-1}$ if and only if $\gamma > 1$. Define $N \times 1$ vector $\Delta\mathbf{K}_\tau$ whose i^{th} element is given by $k_{i,\tau-1} - k_{i,\tau}$ for $\tau = 1, 2, \ldots, T-1$ and define $N \times N$ diagonal matrix $\mathbf{Z}$ whose i^{th} diagonal element is $z_i = \beta \exp(a_i)$. One can show that $\Delta\mathbf{K}_\tau$ can be expressed as follows:

$$\Delta\mathbf{K}_\tau = (\mathbf{I}-\mathbf{A})\mathbf{A}^\tau(\mathbf{I}-\mathbf{HQ})^{-1}[\mathbf{Q}(\mathbf{I}-\mathbf{A}^T)-(\mathbf{I}-\mathbf{HQ})]\mathbf{A}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{1}$$

where $\mathbf{1}$ is a $N \times 1$ vector of ones and $(\mathbf{I}-\mathbf{A})\mathbf{A}^\tau$ is a diagonal matrix whose elements are positive. For $N = 2$, one can show that

$$(\mathbf{I}-\mathbf{HQ})^{-1}[\mathbf{Q}(\mathbf{I}-\mathbf{A}^T)-(\mathbf{I}-\mathbf{HQ})]\mathbf{A}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{1} =$$
$$\frac{1}{c}\left(\frac{z_1}{1-z_1}-\frac{z_2}{1-z_2}\right)\begin{pmatrix}(1+(z_2^T-z_1^T)(1-[\mathbf{Q}]_{1,1}-[\mathbf{Q}]_{2,2})[\mathbf{Q}]_{2,2})(1-z_2^T)([\mathbf{Q}]_{1,1}-1)\\(1+(z_1^T-z_2^T)(1-[\mathbf{Q}]_{1,1}-[\mathbf{Q}]_{2,2})[\mathbf{Q}]_{1,1})(1-z_1^T)(1-[\mathbf{Q}]_{2,2})\end{pmatrix}$$

where c is the determinant of $(\mathbf{I}-\mathbf{HQ})$, hence positive. $[\mathbf{Q}]_{i,j}$ denotes the ij^{th} element of the transition probability matrix, $\mathbf{Q}$. Using the above equation and the fact that $(\mathbf{I}-\mathbf{A})\mathbf{A}^\tau$ is a diagonal matrix whose elements are positive, one can show that the first element of $\Delta\mathbf{K}_\tau$ is positive and the second element is negative if and only if $\gamma > 1$. Hence, the difference between $k_{1,\tau} - k_{2,\tau}$ in absolute value gets smaller as τ approaches 0 if $\gamma neq 1$. This automatically implies the statement in Proposition 6. □

Proof of Proposition 7. The derivative of conditional volatility at time t with respect to any news variable can be written as follows:

$$\partial \text{var}_t(r_{t+1}^*)/\partial u_{.,t} = (\sigma_{d,1}^2-\sigma_{d,2}^2)\frac{\partial \pi_{1,t}}{\partial u_{.,t}} + (m_{1,\tau-1}-m_{2,\tau-1})^2(1-2\pi_{1,t})\frac{\partial \pi_{1,t}}{\partial u_{.,t}}$$

and plugging in the derivative of $\pi_{1,t}$ with respect to the appropriate news variable gives the equations in Proposition 7. □

Proof of Proposition 8. Proofs of (a) and (b) First note that

$$(\sigma_{d,1}^2 - \sigma_{d,2}^2) + (m_{1,\tau-1} - m_{2,\tau-1})^2(1 - 2\pi_{1,t})$$

is positive if and only if $\pi_{1,t} > \pi_1^*$. Then under the conditions of Proposition 8, the reaction of conditional volatility depends on whether $\Delta d_t > \Delta d^*$ or $x_n > x^*$ holds or not.

Proof of (c). If the dividend growth process has the same volatility in both states, i.e. $\sigma_{d,2} = \sigma_{d,1}$, then π_1^* is equal to 0.5. □

Proof of Corollary 2. Proof of (a) Using the definition of $\pi_{1,t}$ and assuming $\sigma_{d,2} > \sigma_{d,1}$, $\pi_{1,t} > 0.5$ if and only if the following holds

$$1 + \tfrac{1-\pi_{1,t}}{\pi_{1,t}} \exp\left(\tfrac{\sigma_{d,2}^2 - \sigma_{d,1}^2}{2\sigma_{d,1}^2\sigma_{d,2}^2}(\Delta d_t - \Delta d^*) - \tfrac{(\mu_{d,1}-\mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)} \right) < 2$$
$$|\Delta d_t - \Delta d^*| < \delta(\pi_{1,t-1})$$

where $\delta(\pi_{1,t-1})$ is given by

$$\delta(\pi_{1,t-1}) = \left(\frac{2\sigma_{d,1}^2\sigma_{d,2}^2}{\sigma_{d,2}^2 - \sigma_{d,1}^2} \left(\log \frac{\pi_{1,t}}{1-\pi_{1,t}} + \frac{(\mu_{d,1}-\mu_{d,2})^2}{2(\sigma_{d,2}^2 - \sigma_{d,1}^2)} \right) \right)^{1/2}$$

Similarly, assuming $\sigma_{d,2} < \sigma_{d,1}$, one can show that $\pi_{1,t} < 0.5$ if and only if $|\Delta d_t - \Delta d^*| < \delta(\pi_{1,t-1})$

Proof of (b) Using the definition of $\pi_{1,t}$ and assuming $\sigma_{d,2} = \sigma_{d,1}$, $\pi_{1,t} > 0.5$ if and only if the following holds

$$1 + \tfrac{1-\pi_{1,t}}{\pi_{1,t}} \exp\left(\tfrac{1}{2\sigma_{d,1}^2}(2(\mu_{d,2} - \mu_{d,1})\Delta d_t + \mu_{d,1}^2 - \mu_{d,2}^2) \right) < 2$$
$$\Delta d_t > \tfrac{\mu_{d,1}+\mu_{d,2}}{2} - \tfrac{\sigma_{d,1}^2}{\mu_{d,1}-\mu_{d,2}} \log \tfrac{\pi_{1,t}}{1-\pi_{1,t}}$$

□

Proof of Proposition 9. The proof of Proposition 9 is very similar to the proof of Proposition 8. Hence, the reader is referred to the proof of Proposition 8. □

Proof of Proposition 10. Proof of (a) If $\sigma_{d,2} < \sigma_{d,1}$, then the magnitude of $(\sigma_{d,1}^2 - \sigma_{d,2}^2) + (m_{1,\tau-1} - m_{2,\tau-1})^2(1 - 2\pi_{1,t})$ decreases as τ approaches to zero since we know that $k_{1,\tau} - k_{2,\tau}$ in absolute value gets smaller as τ approaches.

Proof of (b) If $\sigma_{d,2} > \sigma_{d,1}$, then the magnitude of $(\sigma_{d,1}^2 - \sigma_{d,2}^2) + (m_{1,\tau-1} - m_{2,\tau-1})^2(1 - 2\pi_{1,t})$ decreases as τ approaches to zero if and only if the sign of $(\sigma_{d,1}^2 - \sigma_{d,2}^2) + (m_{1,\tau-1} - m_{2,\tau-1})^2(1 - 2\pi_{1,t})$ (i.e. $\pi_{1,t} > 0.5$) since we know that $k_{1,\tau} - k_{2,\tau}$ in absolute value gets smaller as τ approaches.

Proof of (c) If $\gamma = 1$, then $m_{1,\tau-1} = m_{2,\tau-1}$. Hence, the reaction of conditional volatility does not depend on τ. □

Proof of Corollary 3. This follows directly from the definition of conditional volatility and the fact that $(m_{1,\tau-1} - m_{2,\tau-1})^2$ decreases as τ approaches zero. $\square$